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Spin structures in inhomogeneous fractional quantum Hall systems

Karel Výborný*, Daniela Pfannkuche

I. Institute of Theoretical Physics, University of Hamburg, Jungiusstr. 9, Hamburg D-20355, Germany

Abstract

Formation of domains in highly correlated fractional quantum Hall states is the anticipated explanation of the huge longitudinal magnetoresistance (HLM) phenomenon. Nuclear spins are expected to be responsible for disturbing the homogeneous ground states. We use a finite size model employing exact diagonalization to see how the fractional quantum Hall states respond to magnetic impurities. Results indicate that domains cannot build up when only the incompressible states are involved.

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Keywords: Fractional quantum Hall effect; Correlated states; Magnetic inhomogeneity; Domains

1. Introduction

The discovery of the huge longitudinal magnetoresistance (HLM) phenomenon [1] opened new promising ways to measure indirectly the nuclear spin polarization in GaAs/GaAlAs heterostructures by means of conductivity measurements rather than by e.g. NMR [2]. The HLM has been experimentally studied on high-mobility two-dimensional electron gases in the fractional quantum Hall regime (filling factor $\frac{2}{3}$) where the ground state is known to be spin unpolarized for lower magnetic fields and spin polarized for higher magnetic fields [3]. Although there is a common consensus that domains of these two types of states build up near the transition field, little is known about microscopic processes leading to formation of such domains. We present a microscopical model describing the transition between the unpolarized and fully polarized ground states and study the transient states.

2. Model

Our studies are based on the standard Yoshioka model of two-dimensional homogeneous systems subjected to a perpendicular magnetic field [4] which addresses few-particle states restricted to the lowest Landau level (LLL). Its basic idea is to replace the infinite two-dimensional electron gas by a rectangular primitive cell (of size a by b) with periodic boundary conditions in both directions. The number of possible single-particle states (N_m) becomes finite and hence the few-particle basis (composed of products of the single-particle states) will also have a finite dimension. It can be shown that N_m is the number of magnetic

* Corresponding author.
E-mail address: kvyborny@physnet.uni-hamburg.de (K. Výborný).

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flux quanta passing through the unit cell and thus the filling factor $\nu = N_e/N_m$ can be adjusted by choosing N_e and N_m.

Given the basis we compute matrix elements of the Hamiltonian (we skip the constant one particle term $\frac{1}{2}\hbar\omega$)

$$H_{\mathrm{Coul}} = \frac{e^2}{4\pi\varepsilon}\sum_{i<j}\frac{1}{|r_i - r_j|} \tag{1}$$

and diagonalize it. This yields the exact ground state under the assumption of periodic boundary conditions. Note that H_{Coul} scales with magnetic field as $\sqrt{eB/\hbar} = 1/\ell \propto \sqrt{B}$ since the matrix elements can be evaluated in dimensionless coordinates r_i/ℓ.[1]

We consider three extensions to H_{Coul}: (i) the Zeeman splitting

$$H_{\mathrm{Zeeman}} = \mu_B g B s_z \tag{2}$$

can make two states from the spectrum of H_{Coul} degenerate at suitable B because of different scaling behaviour versus B. Further we take into account an inhomogeneous magnetic field along the z-axis (ii) and along the x-axis (iii) both varying in the x direction

$$H_z = \mu_B g V_z(x) s_z, \quad H_x = \mu_B g V_x(x) s_x \tag{3}$$

with $V_z(x) = B_z \cos 2\pi x/a$, $V_x(x) = B_x \sin 2\pi x/a$. We neglect the effect of the additional magnetic field on the orbital motion (our basis in the LLL is still the one of homogeneous magnetic field; therefore it would be more consistent to speak of H_z rather as of H_{Zeeman} with spatially varying g-factor and keep the magnetic field constant). H_z makes the Zeeman splitting effectively larger in one part of the primitive cell and H_x is necessary for the two crossing states to mix and therefore to allow them to build up a ground state with spatially varying spin.

3. Results

3.1. Homogeneous systems

Fig. 1 shows spectra of systems with six and eight particles at filling factor $\frac{2}{3}$ whose Hamiltonian consists solely of the H_{Coul} part. The states are sorted according to the value of total spin ($[H_{\mathrm{Coul}}, S^2] = 0$), states which differ only by the value of S_z are degenerate. The lowest lying unpolarized state ($|S = 0; J\rangle$) and fully polarized state ($|S = N_e/2; J\rangle$) are—apart from the degeneracy in S_z—triply degenerate in the centre-of-mass x-coordinate J (number three comes from the denominator of $\nu = \frac{2}{3}$, [5]) and are related by magnetic translations in the x direction (preferred direction is given by choice of the gauge). This is well demonstrated by their single-particle densities (Fig. 2). Striking inhomogeneity of the densities is an effect of system's finite size [6] and their (thermal) average is spatially constant to a very good precision with the isotropy between x and y directions recovered, too.

The Zeeman term lifts the degeneracy in S_z and we obtain spectra as shown in Fig. 3. Both ground states $|S = 0; J\rangle$ (for $B \to 0$) and $|S = N_e/2; J\rangle$ (for $B \to \infty$) are separated from the rest of the spectrum by a noticeable gap. In the interim range of B there is a direct crossing between the two ground states (we denote the magnetic field at this transition by B_c). In the following, we will focus on the transition point and will try to induce domains by switching on the inhomogeneity (of the magnetic field).

3.2. Inhomogeneous systems

Intuitive approach suggests that an $S = 0$ state is favourable near to $x = a/2$ and an $S = N_e/2$ state is favourable near to $x = 0$ in a system described by the Hamiltonian $H_{\mathrm{Coul}} + H_{\mathrm{Zeeman}} + H_z$. In spite of this the spectrum (as the one in Fig. 3) remains qualitatively unchanged provided the impurity, H_z is weak. Especially there is still a crossing between the $|S = 0; J\rangle$ and $|S = N_e/2; J\rangle$ ground states and the value of $\langle S\rangle$ for the ground state as a function of magnetic field exhibits a discontinuity at the crossing. The reason for this is simple: the crossing ground states are also eigenstates to S_z with different eigenvalues and since $[H_z, S_z] = 0$ the overlap $\langle S = 0; J|H_z|S = 3; J\rangle$ vanishes and thus there is no mixing between the two states in lowest order perturbation theory. On the other hand, if H_z is strong the original unpolarized ground state cannot be recognized any more (it lies now in the quasi continuum of polarized states) and thus the guide is lost where to look for a domain state.

[1] r_i can be obviously expressed in units of a and b. As stated before, the primitive cell has to contain N_m flux quanta ($ab = 2\pi N_m \ell^2, \ell^2 = \hbar/eB$) for any B. Thus, keeping N_m and $a : b$ constant, $a, b \propto \ell$.

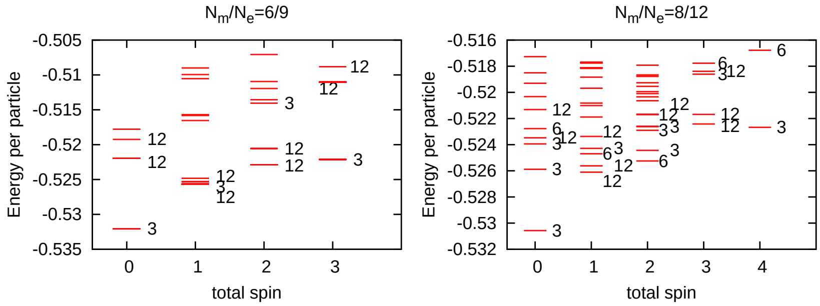

Fig. 1. Spectra of a homogeneous system without Zeeman splitting at $\nu = \frac{2}{3}$ with six and eight particles (all energies throughout this article are in units $e^2/(4\pi\epsilon)$). States are sorted by the value of total spin, degeneracy (neglecting degeneracy in S_z) is indicated by numbers at levels.

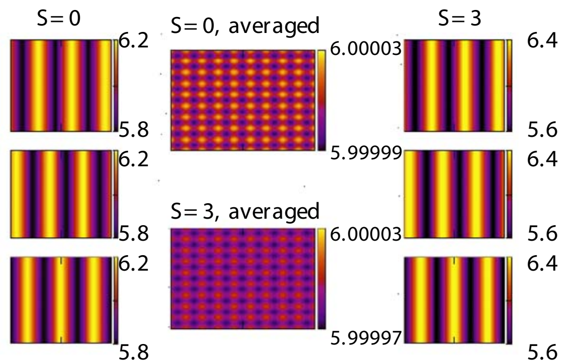

Fig. 2. Single-particle densities in the primitive rectangular cell. The three degenerated lowest lying six-particle states with $S = 0$ and 3 (referring to Fig. 1) are displayed. Their averages are to a very good precision homogeneous (note the scales of the colour axis).

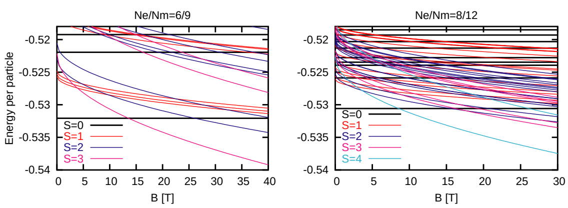

Fig. 3. Spectra of homogeneous systems with six and eight particles ($\nu = \frac{2}{3}$) with Zeeman splitting displayed as a function of magnetic field. There is a direct crossing between the unpolarized ($S = 0$; horizontal line) and fully polarized ($S = 3$ and 4 for six and eight particles, respectively; the steepest line) state, i.e. $B_c \approx 13$ and 8 T for six and eight particles, respectively. The crossing states are separated from the quasicontinuum by a finite gap.

In order to make $|S = 0; J\rangle$ and $|S = N_e/2; J\rangle$ mix we introduce H_x. We then indeed observe an anticrossing at the former position of the crossing and expectation values of S and S_z in the ground state vary smoothly with magnetic field. Owing to the translational invariance of $H_z + H_x$ along y, the centre-of-mass x-coordinate J remains a good quantum number. We thus obtain a triplet of nearly degenerate 'bonding' states at the anticrossing which differ by J. Their single-particle densities and local expectation values of spin (LEVS) are shown in Fig. 4. Averaging densities and LEVS of the three states (as a means of suppressing the finite size effects) we realize that the states really mimic the structure of H_z: states are 'more polarized' close to $x = 0$ and 'less polarized' near $x = a/2$. For the sake of clarity we show how single-particle density and LEVS change when we sweep magnetic field through the ground state (anti)crossing (Fig. 5). Variation of LEVS is considerably larger at $B = B_c$ than at either side of B_c. This resembles a homogeneous unpolarized-domain-homogeneous polarized transition. Variation of $\langle S\rangle$ at $B = B_c$ is however still very weak (about 0.2%). Calculations for six and eight particles show similar behaviour, the variation of $\langle S\rangle$ remains small (see also Fig. 4).

Comparison of inhomogeneities of different strengths (H_z and H_x) shows that H_x does not affect $\langle S\rangle(x)$ much (as soon as $H_x \neq 0$) and its most prominent role is to determine width of the anticrossing. Response of $\langle S\rangle(x)$ (its variations at the anticrossing) to H_z seems to be linear provided H_z is weak.

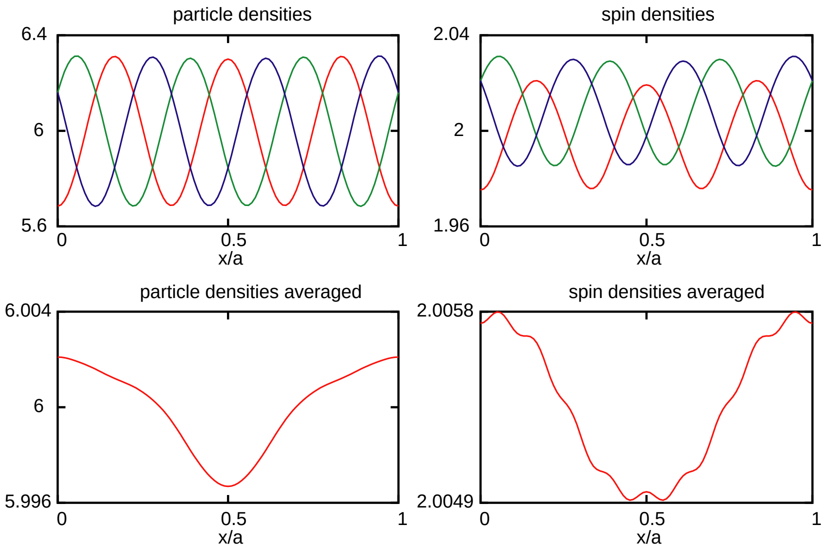

Fig. 4. Single-particle densities and spin in the ground state of an inhomogeneous system at the anticrossing between the fully polarized and unpolarized ground state (system with six electrons). Above the three degenerate states, below average. Inhomogeneous magnetic fields in the z and in the x direction are of the same amplitude. The displayed quantities (as functions of x and y) are nearly constant along y and hence we show them only as functions of x.

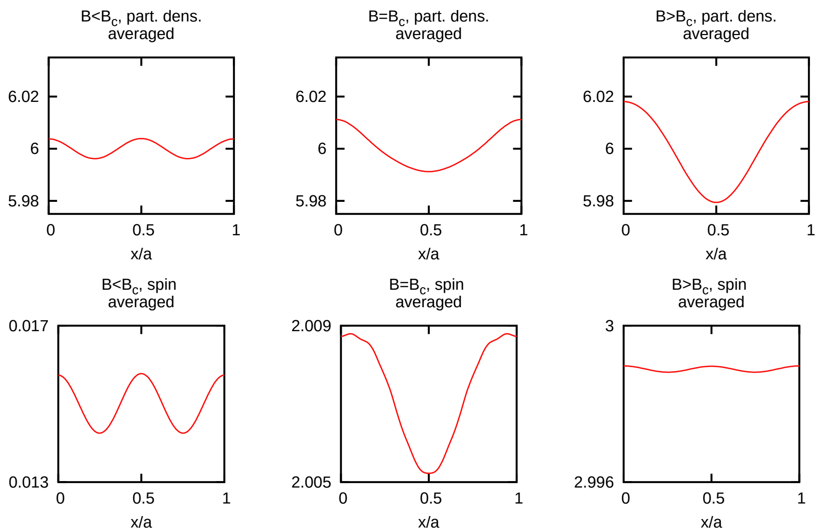

Fig. 5. Single-particle density and spin profile of the ground state at various magnetic fields (system with six electrons). Left to right unpolarized, domain-like and polarized state. Amplitudes of the inhomogeneous part of magnetic field in the z and x direction are in ratio 4:1.

We also looked at how other quantities evolve as we pass the ground state (anti)crossing by sweeping the magnetic field (Fig. 6). The z component of spin looks very similar to the total spin except for the unpolarized state: it is much more homogeneous in S_z than in S. The x component follows the profile of H_x and shows

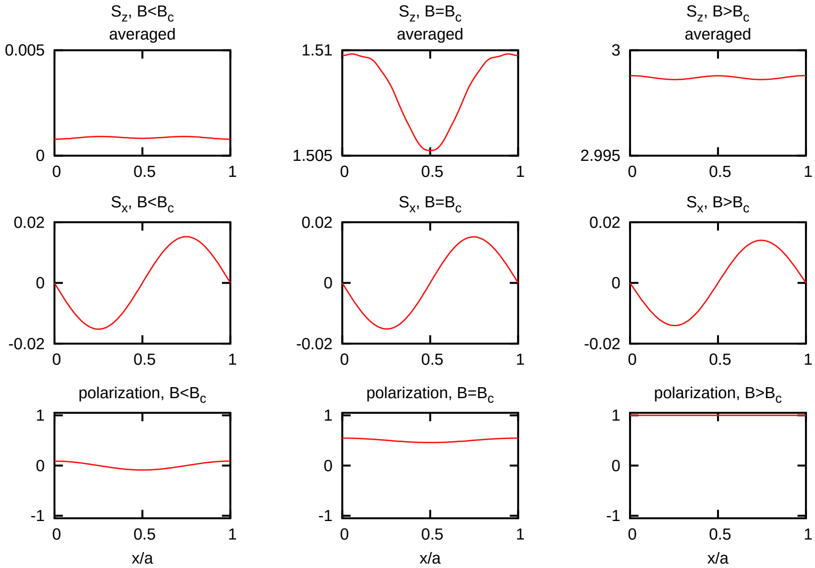

Fig. 6. z component of spin, x component of spin and polarization $(n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$ at $B < B_c$, $B = B_c$ and $B > B_c$ (the same system as in Fig. 5).

no remarkable change; polarization $(n_\uparrow - n_\downarrow)/(n_\uparrow + n_\downarrow)$ transits smoothly from the $B < B_c$ form (unpolarized) to the $B > B_c$ form (polarized) without any new form in between. The last two quantities also do not seem to entail finite size effects.

4. Conclusion

We extended the standard model of the interacting 2D electron gas in a magnetic field by an inhomogeneous Zeeman splitting ($H_{\text{Zeeman}} + H_z$) aiming to study formation of domains of spin polarized and spin unpolarized states. We also added an inhomogeneous inplane magnetic field (H_x) in order to make the transition between the spin polarized and spin unpolarized state continuous rather than step-like. Looking at the spatial variation of the spin (or better its z component) we indeed observed clear signs of a transition from a homogeneous unpolarized to a domain-like and further to a homogeneous polarized state as the magnetic field was varied. The “domains” were rather weak (difference of spin ‘inside’ and ‘outside’ was at best of the order of part in hundred) or in other words spin stiffness is rather large. We believe this hints at incompatibility of the correlations in the polarized and unpolarized incompressible ground state. This would mean that more states (than just the polarized and unpolarized ground state) have to participate in building a state with domain wall—preferably partially polarized states (which turn out to be separated from the ground states by a gap in our model). Other possible explanation, namely smallness of our model system and long-range nature of Coulomb interaction, is less likely since tendency to build domains (spatial variation of spin) in a system with 8 particles is nearly the same as for 6 particles. To confirm this speculation it would be desirable to study the two particle correlation functions of the ground state and also systems with Coulomb interaction replaced by a short-range interaction.

We would also like to draw our attention to the behaviour of the x component of spin. Presence of inhomogeneous inplane magnetic field is essential to form a transient state between the polarized and unpolarized ground states. On the other hand, spatial variations of x and z component of spin are rather decoupled provided H_x is weaker than H_z (there is no sign of transition in S_x around the anticrossing).

Acknowledgements

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