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SPIN STRUCTURES IN INHOMOGENEOUS FRACTIONAL QUANTUM HALL SYSTEMS

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Transitions between spin polarized and spin singlet incompressible ground state of quantum Hall systems at filling factor 2/3 are studied by means of exact diagonalization with eight electrons. We observe a stable exactly half–polarized state becoming the absolute ground state around the transition point. This might be a candidate for the anomaly observed during the transition in optical experiments. The state reacts strongly to magnetic inhomogeneities but it prefers stripe–like spin structures to formation of domains.

Keywords: Quantum Hall ferromagnet; fractional quantum Hall effect; magnetic impurity.

The huge longitudinal magnetoresistance (HLM) effect[1] attracted some attention to fractional quantum Hall systems at filling factor $\nu = 2/3$ and its two competing incompressible ground states (GS): the spin-singlet and the polarized one. By adjusting the ration between Zeeman and Coulomb energy (via B, i.e. electron density) these two GSs can be brought to coincidence and the system may turn into a quantum Hall ferromagnet (QHF) in the fractional regime. Phenomena resembling ferromagnetism (hysteresis, Barkhausen jumps[1]) were indeed measured and it was suggested that domains form in the system when the two GSs are degenerate. On the other hand, optical measurements of electron system polarization γ at $\nu = \frac{2}{3}$ and varying B showed signs of a stable transition state manifested by plateaus in $\gamma(B)$ at exactly[7] $\gamma = 0.5$. This motivated us to investigate the system at the transition theoretically using a microscopic model.

1. How to Attack the Problem — the Model

The standard Yoshioka model[3] replaces the infinite plane by a rectangular primitive cell (of size a by b) with periodic boundary conditions (PBC) in both directions and restricts the one-particle states in it to the lowest Landau level (LLL); the latter step quenches the kinetic energy into an irrelevant constant ($\frac{1}{2}\hbar\omega_c$). For a given number of electrons N_e and filling factor ν this allows only a finite number of N_e–

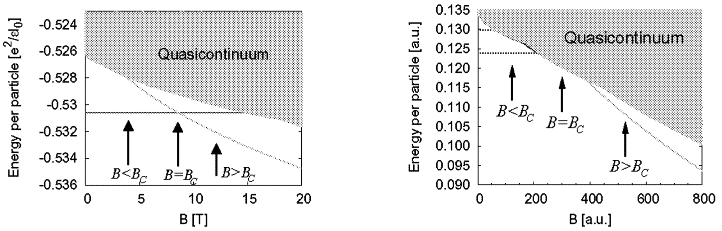

Fig. 1. Energy versus magnetic field of an eight electron homogeneous system with Zeeman term. Left: Coulomb interaction; direct transition from the singlet ($B < B_C$) incompressible GS to the polarized incompressible GS ($B > B_C$), quasicontinuum is well above the crossing. Right: short–range interaction; a new exactly half–polarized state (on the edge of quasicontinuum) is the GS in the transition region.

particle states. The choice of the basis is thus the principal approximation whereas the Hamiltonian can be taken in its full complexity. Its matrix is then evaluated and diagonalized.

We will first consider the Coulomb interaction H_{Coul} and replace it later by a short–range one H_{hc} (inspired by Rezayi[4])

$$H_{\text{Coul}} = \frac{e^2}{4\pi\epsilon}\sum_{i<j}\frac{1}{|r_i - r_j|} = \sum_{i<j}\sum_{k} V_k^C P_{ij}^k\,; \qquad H_{\text{hc}} = \sum_{i<j} V_0^C P_{ij}^0 + V_1^C P_{ij}^1\,. \quad (1)$$

Here P_{ij}^0, P_{ij}^1, ...are projectors onto two-particle (i and j) eigenstates (restricted to the LLL) of an arbitrary interaction described by radial potential. They are sorted according to the mean distance of the particles (in increasing order). Our H_{hc} thus simply takes the Coulomb interaction and skips all but the 'on-site' (P^0) and 'nearest-neighbor' (P^1) interaction terms.

We extend this basic model by (i) the Zeeman term which can bring two states of H_{Coul} with different total S_z to a coincidence owing to its different scaling with B and (ii) a magnetic inhomogeneity (MI; extra magnetic field parallel to B):

$$H_{\text{Zeeman}} = \mu_B g B \sum_i (S_z^i/\hbar)\,, \qquad H_{\text{MI}} = \mu_B g \sum_i B_{MI}(x_i)(S_z^i/\hbar)\,.$$

Particular forms of $B_{MI}(x)$ will be mentioned later. The effect of this additional magnetic field on the orbital motion is neglected — our basis in the LLL corresponds still to a homogeneous magnetic field and it would thus be more consistent to speak of $H_{\text{Zeeman}} + H_{\text{MI}}$ rather as of H_{Zeeman} with spatially varying g–factor and keep the magnetic field constant.

2. Ground state at the transition

Let us look at a homogeneous system with Zeeman energy and change the magnetic field while keeping the filling factor constant (i.e. change the electron density and thus also ratio between Zeeman and Coulomb energy). A direct transition from the incompressible (i.e. gapped) singlet GS to the incompressible polarized GS is observed (Fig. 1 left). This happens owing to different scaling of H_{Coul} and H_{hc} with

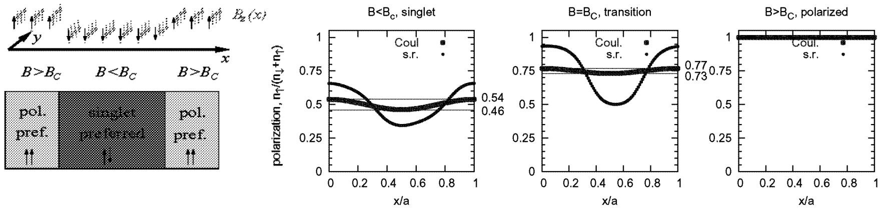

Fig. 2. Response of the ground state (singlet, at the transition and polarized) for $H_{\rm Coul}$ and $H_{\rm hc}$ to magnetic inhomogeneity of the profile shown on the left (supporting domain formation: domain structure expected for uncorrelated electrons shown on left). For Coulomb–interacting system monotonous transition is observed (polarization variation at $B = B_C$ is half the variation for the singlet GS); the transition state (at $B = B_C$) in short–range–interacting systems might form domains. Strength of the MI is 6% and 25% of the singlet GS gap for the Coulomb and short–range interaction, respectively.

B. Now, switch on an inhomogeneity $B_{MI} \propto \text{sgn}\ \cos(x/a)$ (Fig. 2 left) and tune its strength to 10% of the gap (of the singlet GS). We find that response of the GS (at the transition point) to such MI is weak (polarization varies by at most 10%, while non-correlated electrons are expected to form polarized and unpolarized domains with variation 50%); moreover, even the incompressible singlet GS shows a stronger response under these conditions (Fig. 2; details in our former work[6]). Briefly, we observe *no pronounced spin structures* emerging in the GS in the moment when we pass through the transition point $B = B_C$ in this case; even in presence of MI the system tries to remain homogeneous.

The short–range model also predicts a transition between the two types of ground states (and preserves the nature they have for Coulomb interaction as can be seen from electron–electron correlation functions). However, the transition is not direct, Fig. 1 right: in a finite interval of magnetic fields the absolute GS is another state having $S = N_e/4$ (one half of the full polarization). This scheme of GS evolution $S = 0 \to N_e/4 \to N_e/2$ seems not to be bound to hard core systems and is supported by thermodynamic extrapolation of GS energies based on calculations with Coulomb interaction in spherical geometry[5]; concisely, systems with short–range interaction tend to arrive at the thermodynamic limit faster. We would like to stress that this half–polarized ground state (HPGS) differs considerably from the polarized and singlet incompressible GSs; in our eight–particle calculation it is a quadruplet of states close to the quasicontinuum. Following observations can be done in comparison to the Coulomb systems. (i) Short–range interaction systems sustain stronger MI (while keeping the scheme $S = 0 \to N_e/4 \to N_e/2$): partly owing to larger incompressible gaps and partly because of a larger difference between the energy of $H_{\rm MI}$ in the singlet and the polarized GS. (ii) The HPGS reacts more sensitively to an MI than the singlet GS and also more sensitively (MI 25% of the singlet gap produces polarization variation over 40%) than the GS in Coulomb system at $B = B_C$. This suggests that spin structures may evolve in the HPGS.

Next we applied an MI with $B_{MI}(x) = \delta(x - x_0)$ (plus the PBC; i.e. 'one stripe'). Surprisingly, polarization of the HPGS acquires $a/2$–periodic form ('two

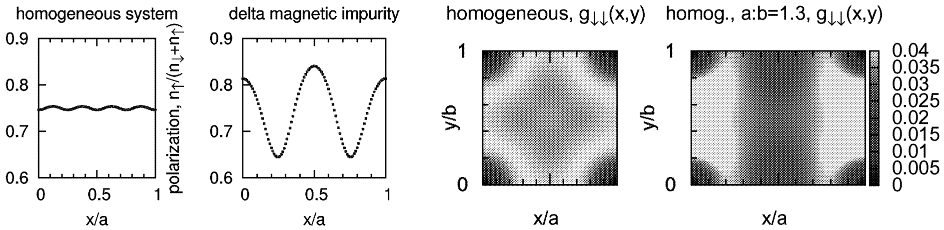

Fig. 3. Reaction of the half–polarized ($S_z = N_e/4$) state to symmetry breaking stimuli. Left: delta magnetic impurity $B_{MI}(x) = \delta(x - 0.5a)$ induces two stripes (along y) in polarization. Right: stretching the primitive cell along x direction induces even in homogeneous system a stripe along y (spin–spin correlation function $g_{\downarrow\downarrow}(r) = \langle \delta(r_1 - r_2 - r)\delta_{\sigma_1\downarrow}\delta_{\sigma_2\downarrow} \rangle$ is shown).

stripes'), Fig. 3. Also slightly stretching the primitive cell of the homogeneous system along the x direction makes the spin down (minority) electrons align into a stripe in the y direction as seen in the spin–spin correlation function (Fig. 3). We also studied low–lying excited states in the $S = N_e/4$ sector and found many other states tending to build stripes. It is worth of noticing that while $H_{\rm MI}$ of the form $B_{MI}(x) \propto \text{sgn}\ \cos(x/a)$ affects the HPGS and also the singlet GS noticeably, the delta stripe MI has nearly no impact on the latter state (response in the HPGS is by an order of magnitude larger than in the singlet GS). It strongly suggests that the stripe–like spin structures are naturally contained already in homogeneous HPGS and the delta stripe MI is the right symmetry–breaking agent to uncover them.

In conclusion, within our studies with eight electron systems, we found no pronounced tendency to domain formation at the transition between the two incompressible ground states with different spin polarization at $\nu = 2/3$. Instead, a new stable exactly half–polarized state becomes the ground state near the transition point which we expect to be gapless for larger systems. It responds more strongly to magnetic inhomogeneities and it has a tendency to form stripe–like structures when symmetry of the system is disturbed. To unveal the nature of this state in infinite systems studies with more than eight particles will be essential. We acknowledge support from GrK235 and thank Benjamin Krüger for his help with numerical calculations.

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