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Effect of Disorder on Spin and Charge Excitations in the Fractional Quantum Hall Effect

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A simple model of disorder in fractional quantum Hall systems is combined with the standard exact diagonalisation technique. Electron-density--dependent gaps at filling factors $1/3, 2/3, 2/5$, and $3/5$ measured by activated transport can then be fitted with a single reasonable value of d which has the meaning of the separation of ionized donors from the quasi-2D electron gas.

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1. Introduction

Incompressibility gaps of the fractional quantum Hall (FQH) states observed in experiments are usually smaller than the ones calculated in ideal systems. Some sources of the gap reduction can be estimated rather precisely (a finite thickness of the quasi-2D electron gas, Landau level mixing) but one thing traditionally remains poorly explored: the disorder. Stemming mostly from remote ionized donors producing a long-range electric potential independent of magnetic field B, some authors accounted for the disorder by a negative additive constant E_d of the order of several K. The transport activation energy as a function of B, i.e. the electron density n, when the filling factor $\nu = n/(eB/h)$ is fixed, can be fitted by $\Delta(B)$ [K] $= 50E_\mathrm{C}\sqrt{B[\mathrm{T}]} + E_d$ with a reasonable success [1]. Here, the Coulomb energy E_C of the corresponding excitation can be calculated by an exact diagonalization including the gap reduction mechanisms mentioned above, and the only fitting parameter remains $|E_d|$ [2], which turns out to be smaller for higher mobility samples in accord with common sense. It also used to be popular in earlier works to take $E_d = 0$ and assume the disorder to reduce the value of E_C directly [3]. However, it is difficult to explain in this way why Δ vanishes for nonzero B, and we are also not aware of any detailed microscopic view supporting this model.

Microscopical models of disorder employed the exact diagonalization (ED) with a single impurity [4–6], a composite fermion picture [7], and topologically-detected localized many-body states in the ED with many impurities [8]. While substantially extending the theoretical understanding of the notion of incompressibility in the FQH regime, the link to the experimental $\Delta(B)$ was not straightforward. The last mentioned work, however, is worth of special attention, not only for its ingenious technique but also because it shows the way from the disorder-reduced gap down to the experimentator-fancied quantity — the mobility. In this communication we describe a simple single-impurity ED model which translates the disorder-induced gap reduction, or better the critical field B_0 at which the incompressibility is switched on into another experimentally attainable quantity: distance d to remote ionized donors. It provides a basis to understand experiments on GaAs/GaAlAs heterostructures described in Ref. [1].

2. Model of the disorder

The exact diagonalization can handle arbitrary many- or single-body Hamiltonians at the cost of the restraint to small systems. We use a sphere with $\approx$ 10 electrons and

$$H = \frac{e^2}{4\pi\varepsilon}\sum_{i<j}\frac{1}{|r_i - r_j|} + \sum_i V(r_i), \quad V_\mathrm{Coul}(r) = \frac{-e^2}{4\pi\varepsilon}(|r|^2 + d^2)^{-1/2} \qquad (1)$$

with the constraint to the lowest Landau level. When the energies (E_C) are expressed in the Coulomb units $e^2/(4\pi\varepsilon\ell_0)$ and the filling factor is fixed, the model $V = V_\mathrm{Coul}$ depends only on a single parameter d/ℓ_0. It has the meaning of an effective distance of a single remote ionized donor compared with the magnetic length $\ell_0 = \sqrt{\hbar/eB}$. The crucial feature of this model is that with decreasing magnetic field and constant d the effective impurity distance d/ℓ_0 diminishes and the effect of the impurity becomes stronger. The charged-donor-model is not exceptional in this respect, an alternative, $V_\mathrm{Gauss}(r) = V_0\exp(-r^2/\sigma^2)$, will be mentioned in the discussion.

What has just been described is readily seen in Fig. 1a showing the full spectra of $\nu = 2/5$ with ten electrons. The many-body states can be distinguished

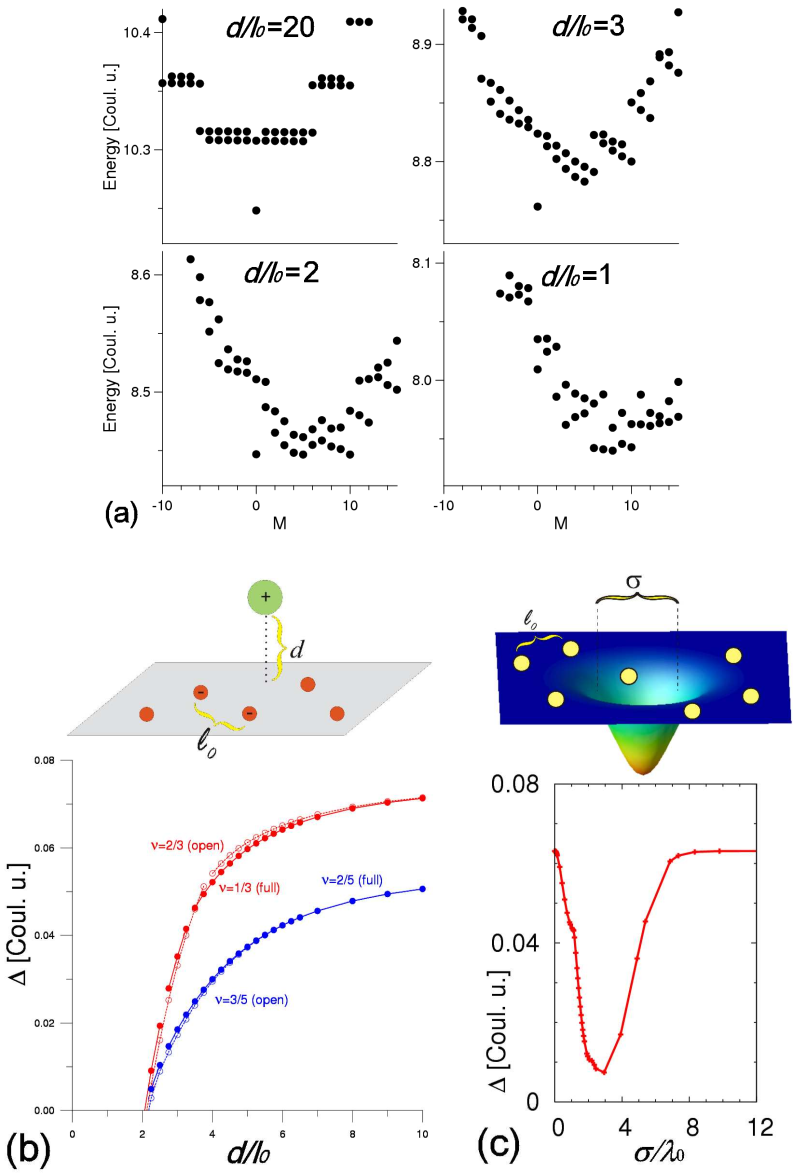

Fig. 1. (a) In the presence of a charged impurity, the spectra of a $\nu = 2/5$ system depend on the single parameter d/ℓ_0. (b) The gap gradually decreases with lowering d/ℓ_0 and vanishes near $d/\ell_0 = 2$ both for $\nu = 2/5$ and $\nu = 1/3$. (c) A different model of the impurity, $V_{\text{Gauss}}(r)$ with constant height and variable width σ, leads to a similar result: the gap gradually decreases with lowering σ/ℓ_0.

by the z-component of the angular momentum (M) because the impurity placed at the north pole of the sphere retains the axial symmetry. The homogeneity of the system ($d/\ell_0 = 20$) is first broken, the gap decreases ($d/\ell_0 = 3$) and finally vanishes at $d/\ell_0 \approx 2$. The sizes of the gap for this one and other filling factors are shown in Fig. 1b. While a similarity between $\frac{2}{5}/\frac{3}{5}$ or $\frac{1}{3}/\frac{2}{3}$ is not surprising (nearly particle–hole symmetry), it is remarkable that gaps of the 1/3 and 2/5 states vanish at almost the same value of $d/\ell_0 \approx 2$.

The data in Fig. 1b can be recalculated into the experimentally measured gap $\Delta(B)$ and with a single unknown parameter d. We now refer to measurements reported in previous work [1] but now extended from $\nu = 1/3$ to some other filling factors, Fig. 2a. Trying to fit the critical field B_0 of the incompressibility onset (e.g. $B_0 \approx 4.0$ T for the 2/5 state in Fig. 2a) the $d/\ell_0 = 2$ condition implies $d \approx 25$ nm. Nevertheless, we fail to reproduce the notable difference between 2/5 and 3/5 gaps, Fig. 2a, in this manner.

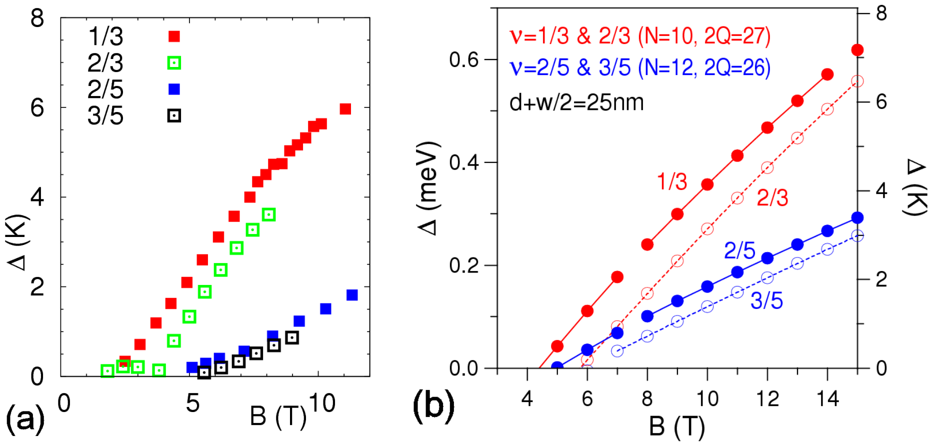

Fig. 2. (a) Gaps at various filling factors determined experimentally from the activated transport (details in [1]). (b) Calculated no spin-flip gaps in the presence of a single charged impurity.

At this point, we unsheathe the finite thickness of the quasi-2D system which effectively softens the electron–electron interaction on short distances. While this circumstance somehow diminishes the gap, the crucial point is that the thickness depends on the electron density, and thus the filling factor, $w \approx 30\ \text{nm}/(B\nu)^{1/3}$ (Eq. (4) in [1]): similar to a parallel-plate capacitor, the higher n, the stronger the electric field and hence the more squeezed the electron gas. The otherwise almost particle–hole (PH) symmetric states thus become notably split, Fig. 2b.

3. Discussion

Although we have suggested above that the gaps, which vanish at nonzero B_0, can be understood in terms of remote charged impurities, it is not the sole alternative. The magnetic length can be compared not only with a "vertical" length scale (d) but also with a lateral scale, σ, and the result is qualitatively similar. As the magnetic field and σ/ℓ_0 decreases, while V_0 is constant, the gap is reduced and may close. The benchmark calculation with five electrons at $\nu = 1/3$ on a torus, Fig. 1c, suggests that this is the case for $V_0 \gtrsim 0.5$ in the Coulomb units. Physically, such situation could occur as a consequence of host lattice defects or interface/surface roughness and if the relevant potentials were sufficiently short--ranged, the gap could even restore at yet lower magnetic fields as it is exemplified at $3\ell_0 > \sigma$ in Fig. 1c. A more detailed discussion of this model is presented elsewhere and we should only keep in mind that the results in Fig. 2 cannot be taken as an evidence for a particular type of disorder in the experiment.

So far we have considered only excitations without spin flips. However, spin flips may occur [1], in particular for $\nu = 1/3$ and $B \lesssim 10$ T for the sample shown in Fig. 2a. Given this circumstance, can we rely on explaining parallel courses of $\Delta(B)$ only using the disorder argument as in Fig. 2b?

The first step to understand this are the comparable energies for spin-waves at $\frac{1}{3}$ and $\frac{2}{3}$ (≈ 0.045, and 0.035 Coulomb units in the long-wave vector limit) in disorder-free systems. Because of the absent particle–hole symmetry of not-fully--polarized ν and $1 - \nu$ states [6], this is a nontrivial result.

We have not performed any calculations with disorder and spin flips yet. The disorder will affect energies of different excitations differently, depending on the many-body correlations manifested in the size and structure of relevant quasiparticles. One can nevertheless speculate that the differences will be small because the charge densities of for example quasielectron with or without spin flip [9] do not interact too differently with $V(r)$ of Eq. (1).

In conclusion, the presented model of the disorder provides a possibility to interpret the measured onset of incompressibility gap at different filling factors using a single disorder-related parameter (remote donors to the heterointerface distance, $d \approx 15$ nm). This model could be valid also for spin-flip excitations, likely to occur in the experiments, but this remains to be tested numerically.

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