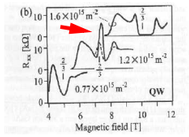
Spin structures in inhomogeneous fractional quantum Hall systems

KAREL VÝBORNÝ AND DANIELA PFANNKUCHE
I. Institute of Theoretical Physics, University of Hamburg, Jungiusstr. 9, 20355 Hamburg, Germany

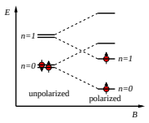
Fractional quantum Hall system, $\nu = 2/3$

Longitudinal magnetoresistance [1]

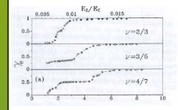


Ground states at various B's

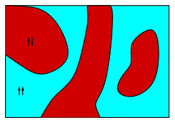
Composite fermion picture



Polarization [2]



- two concurring ground states,
  - fully polarized ($B \to \infty$)
  - spin–singlet ($B \to 0$)
- at transition: quantum Hall ferromagnet?

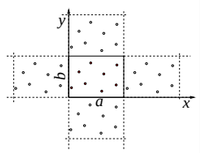


- scattering on domain walls ⇒ enhanced resistance?
- stable half–polarized state?

What happens at the transition?

Model

- rectangle + periodic boundary conditions with $N \approx 8$ electrons
- lowest Landau level
- exact diagonalization

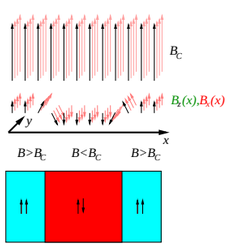


Yoshioka, PRB **29**, 6833 (1984)
Zhang, Chakraborty, PRB **30**, 7320 (1984)

$$H = \frac{e^2}{4\pi\varepsilon}\sum_{i<j}\frac{1}{|r_i - r_j|} + \underbrace{g\mu_B B\sum_i \sigma_i^z}_{\text{Zeeman}} + \underbrace{g\mu_B\sum_i B_z(x)\sigma_i^z}_{\text{spatially varying Zeeman}} + \underbrace{g\mu_B\sum_i B_x(x)\sigma_i^x}_{\text{in-plane inhomogeneity}}$$

- (homogeneous) Zeeman term: brings the two incompressible ground states to degeneracy
- third term: attempt to enforce domains
- fourth term: mixing the two incompressible ground states (over the gap)

Profile of inhomogeneities

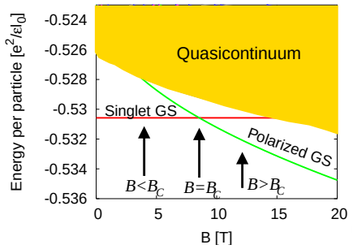


Additional magnetic field

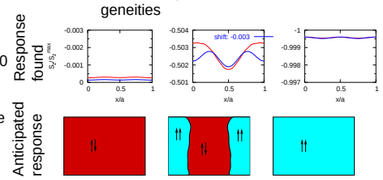
- varying in the x direction
- constant along y
- we expect it to support a domain structure (half of system polarized, half of system unpolarized)

Coulomb interaction: weak response

Homogeneous system + Zeeman term ⟹ inhomogeneity

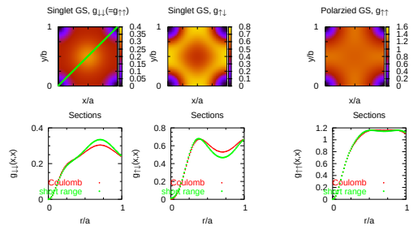


- response (polarization) in the GS changes gradually as we pass $B = B_C$
- quite weak inhomogeneities (10% of the singlet GS gap) destroy the scheme $S = 0 \to S = N_e/2$
- weak response to inhomogeneities



- direct transition between two incompressible ground states (gap); $S = 0 \to S = N_e/2$
- gap over the crossing

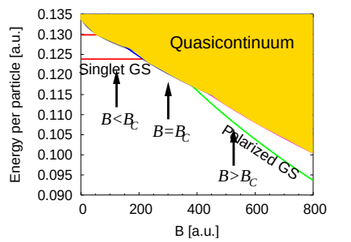
Spin-spin correlation functions for the incompressible ground states



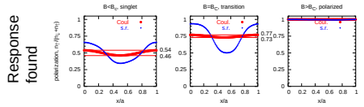
- $g_{\downarrow\downarrow}(\vec{r}) = \langle\delta(\vec{r}_1 - \vec{r}_2 - \vec{r})\delta_{\sigma_1\downarrow}\delta_{\sigma_2\downarrow}\rangle$
- the singlet and the polarized GS are quite different
- the GS's for Coulomb and short–range interactions are nearly the same

Short–range interaction: stronger response

Homogeneous system + Zeeman term ⟹ inhomogeneity



- more sensitive to inhomogeneities; but the scheme $S = 0 \to S = N_e/4 \to S = N_e/2$ more robust (compared to Coulomb)
- transition not gradual (polarization makes jumps during B-sweep)



- transition state between the two incompressible GS's; $S = 0 \to S = N_e/4 \to S = N_e/2$
- this scheme is very likely to persist in the thermodynamic limit (also with Coulomb int.)

Summary

Transition between $S = 0$ and $S = N_e/2$ at $\nu = 2/3$

- if only the two ground states (the spin–singlet one and the polarized one) cross, i.e. there is a gap over the GS in the transition point, no pronounced spin structures can be achieved by magnetic inhomogeneities
- in the thermodynamical limit there seems to be an exactly half–polarized and (probably) gapless state at the transition point

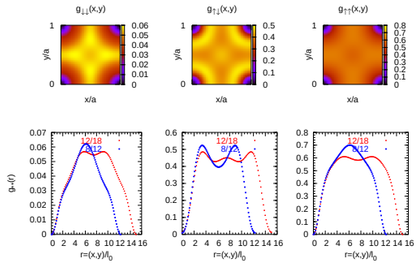
The half–polarized transient state:

- is different to the both incompressible GS's but with some similarities to the singlet GS
- it can be formed into a domain–like form by magnetic impurity of suitable form
- suggests spin pairing ($\uparrow - \downarrow$)

Half–polarized states ($S = N_e/4$)

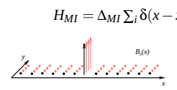
In homogeneous system without Zeeman energy

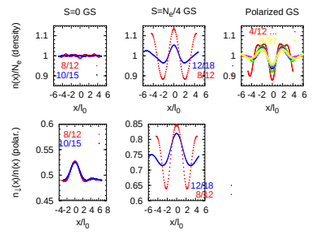
- $g_{\uparrow\downarrow}(r)$ similar to the $S = 0$ incompressible GS: maximum around $r \approx 3.2 l_0$; $\uparrow - \downarrow$ pairing?
- $g_{\uparrow\uparrow}(r)$ and $g_{\downarrow\downarrow}(r)$ have a shoulder around $r \approx 2 l_0$ (like the $S = 0$ GS)



What are its intrinsic spin structures? Apply magnetic impurity $\propto \delta(x - x_0)$ ...

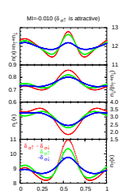
$$H_{MI} = \Delta_{MI}\sum_i \delta(x - x_i)\sigma_z^i$$





- $S = 0$: density very stiff (compared to polarized GS), nonzero polarizability
- $S = N_e/4$: strong response both in density and polarization

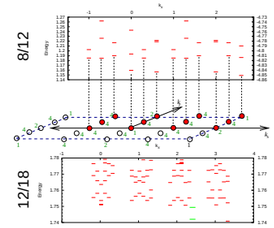
$S = N_e/4$ and H_{MI}: details



$S = N_e/4$ GS a 1 : 1 mixture of $S = 0$ and polarized GS's?

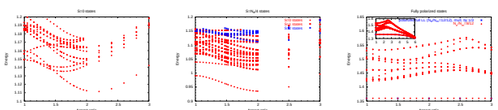
- $n_\downarrow$ responds to $H_{MI} \propto \delta_{\sigma\uparrow}$ similarly like $n_\uparrow$ responds to $\delta_{\sigma\downarrow}$
- $n_\uparrow$ responds to $\delta_{\sigma\uparrow}$ twice as strong than to $\delta_{\sigma\downarrow}$

Spectra: $S = N_e/4$ GS



Stretch the primitive cell along x in homogeneous system

- breaks x-y symmetry
- $S = 0$ more sensitive to aspect ratio change
- $S = N_e/4$ GS has similar $E(a:b)$ like $S = 0$ GS



References

[1] S. Kronmüller et al., *Phys. Rev. Lett.*, **81**, 2526 (1998), J. Smet et al., *Nature*, **415**, 281 (2002), S. Kraus et al., *Phys. Rev. Lett.*, **89**, 266801 (2002); Hashimoto et al., *Phys. Rev. Lett.*, **88**, 176601 (2002).

[2] I.V. Kukushkin et al., *Phys. Rev. Lett.*, **82**, 3665 (1999)

[3] D. Yoshioka, *Phys. Rev. B*, **29**, 6833 (1984); T. Chakraborty, *Surf. Sci.*, **229**, 16 (1990); E. Rezayi, *Phys. Rev. B*, **36**, 5454 (1987)