

# Incompressibility of fractional quantum Hall states

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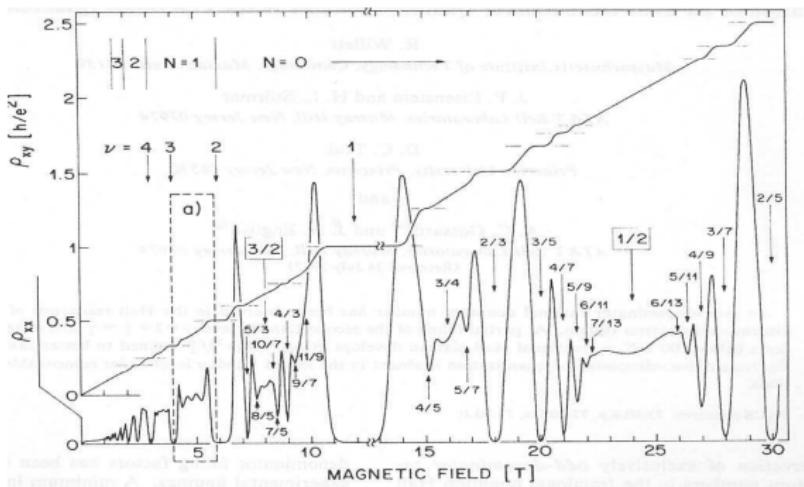
1st Institute of Theoretical Physics  
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and

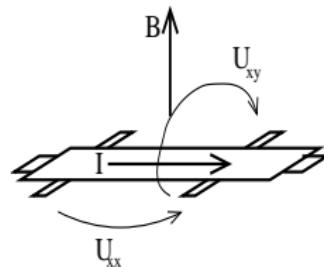
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# Fractional quantum Hall effect



Willett *et al.*, PRL 59, 1776



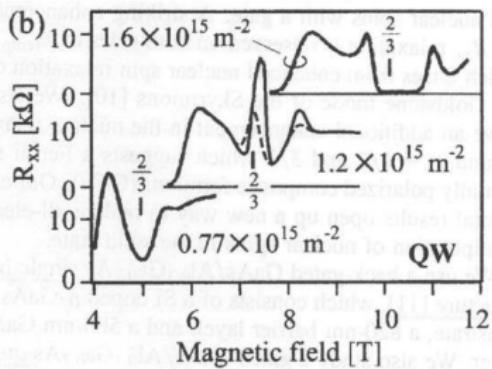
$R_{xx} \rightarrow 0 \Rightarrow$  incompr.  
GS

Incompressible state? What does it mean in terms of response to

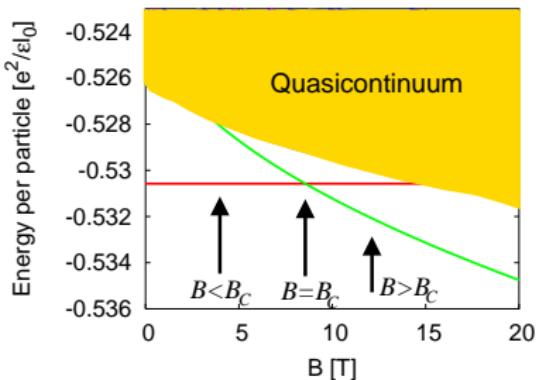
- a non-magnetic impurity
- a magnetic impurity

# Filling factor 2/3: two incompressible ground states

Longitudinal magnetoresistance [1]



Exact diagonalization (ED) spectrum,  $\nu = \frac{2}{3}$

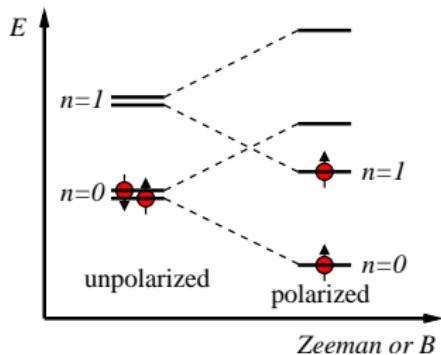


Two competing ground states:

- polarized ( $\uparrow\uparrow$ )
- singlet ( $\uparrow\downarrow$ )

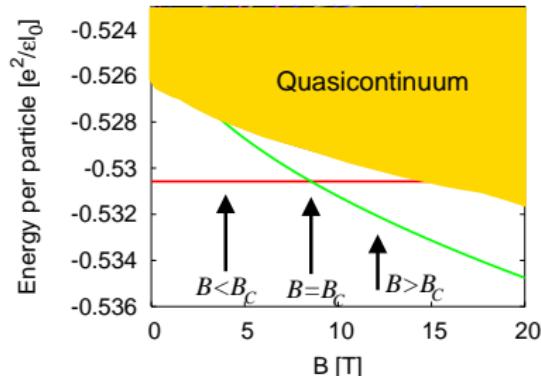
[1] Hashimoto et al., *Phys. Rev. Lett.*, **88**, 176601 (2002)

## Composite fermion picture, $\nu_{CF} = 2$



- $B \rightarrow \infty$ :  $(n = 0, \uparrow), (n = 1, \uparrow) \Rightarrow$
- $B \rightarrow 0$ :  $(n = 0, \uparrow), (n = 0, \downarrow) \Rightarrow$

## Exact diagonalization spectrum, $\nu = \frac{2}{3}$

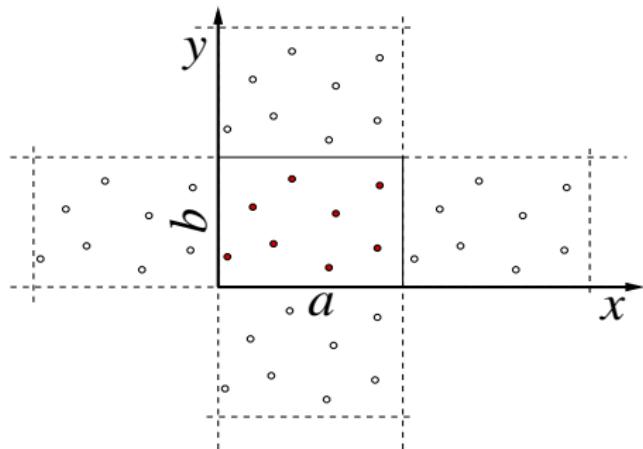


- **polarized (↑↑)**
- **singlet (↑↓)**

# Model

- rectangle + periodic boundary conditions with  $N \approx 10$  electrons
- lowest Landau level
- exact diagonalization

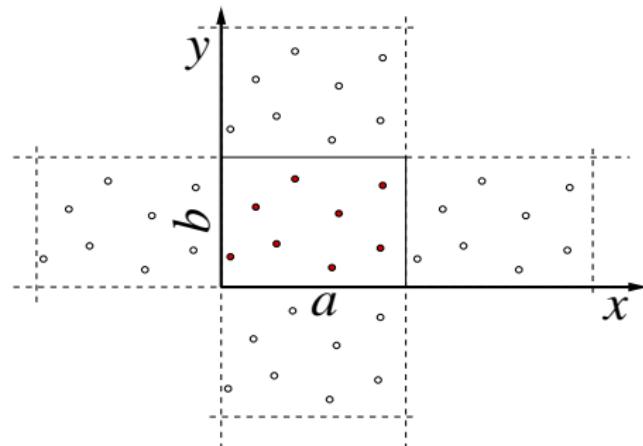
Yoshioka, PRB **29**, 6833 (1984)  
Zhang, Chakraborty, PRB **30**, 7320 (1984)



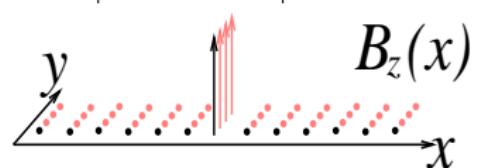
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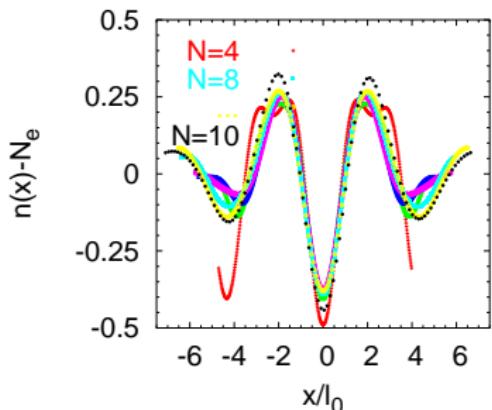


$$H = \frac{e^2}{4\pi\varepsilon} \sum_{i < j} \frac{1}{|r_i - r_j|} + g\mu_B \sum_i B_z(x_i) \sigma_i^z$$



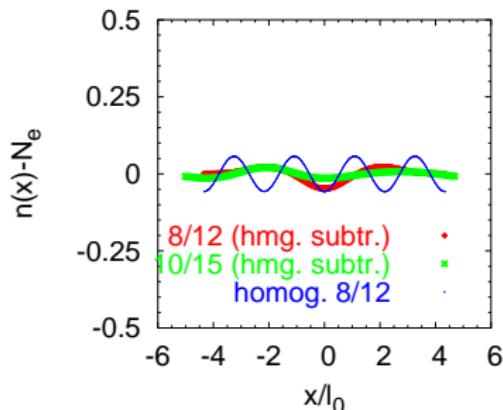
- magnetic impurity (spatially varying Zeeman)
- $\sigma_i^z \rightarrow 1$ : non-magnetic impurity

# Non-magnetic impurity ( $\nu = \frac{2}{3}$ , ground states)



The polarized state

- strong response
- intrinsic length scale (first max.)
- 'incompressible' on length scales  
 $> 10l_0$

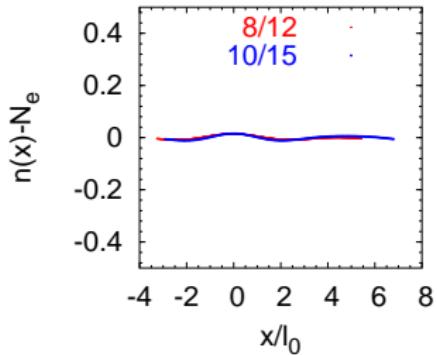


The singlet state

- weak response

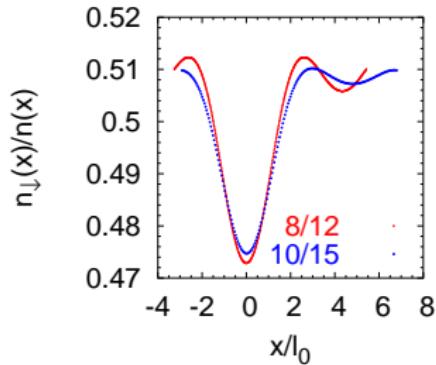
# Magnetic impurity ( $\nu = \frac{2}{3}$ , singlet ground state)

Density



- weak response again

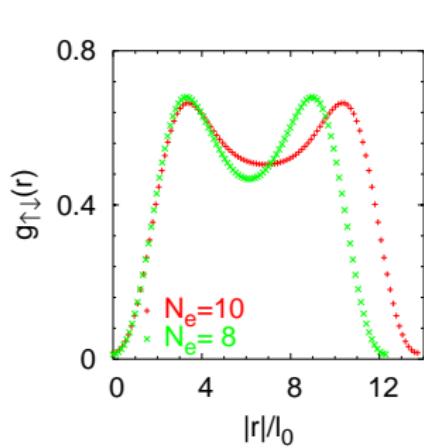
Polarization



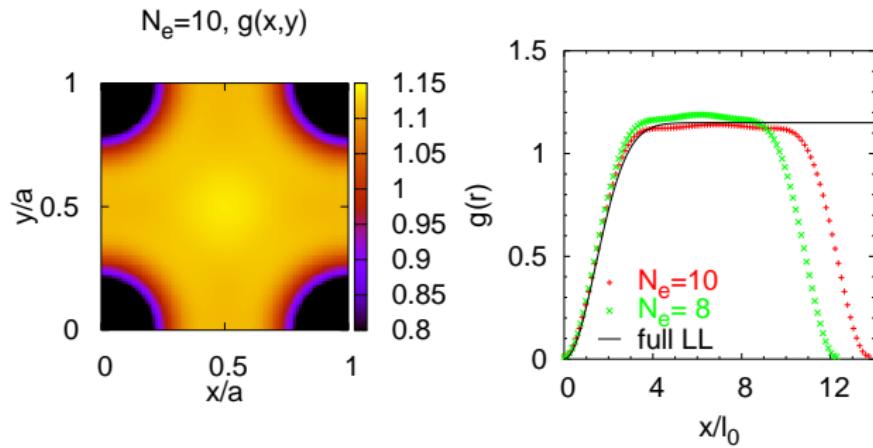
- easily polarizable
- 'compressible'
- no intrinsic length scale

# The singlet ground state: inner structure

Density-density correlation functions  $g(r) = \langle \delta(r_i - r_j - r) \rangle$



$g_{\uparrow\downarrow}(r)$  correlation  
•  $\uparrow$ - $\downarrow$  pairing



Spin-unresolved correlation  
•  $\nu = 1$  incompressible liquid  
of  $\uparrow$ - $\downarrow$  pairs

# Conclusions

- $\nu = 2/3$  fractional quantum Hall states studied by exact diagonalization
- spin-singlet and spin-polarized ground states
- response to non-mag. impurity:  $S = 0$  GS much more rigid
- magnetic impurity:  $S = 0$  GS easily polarizable
- $S = 0$  GS:  $\nu = 1$  liquid of  $\uparrow - \downarrow$  pairs?