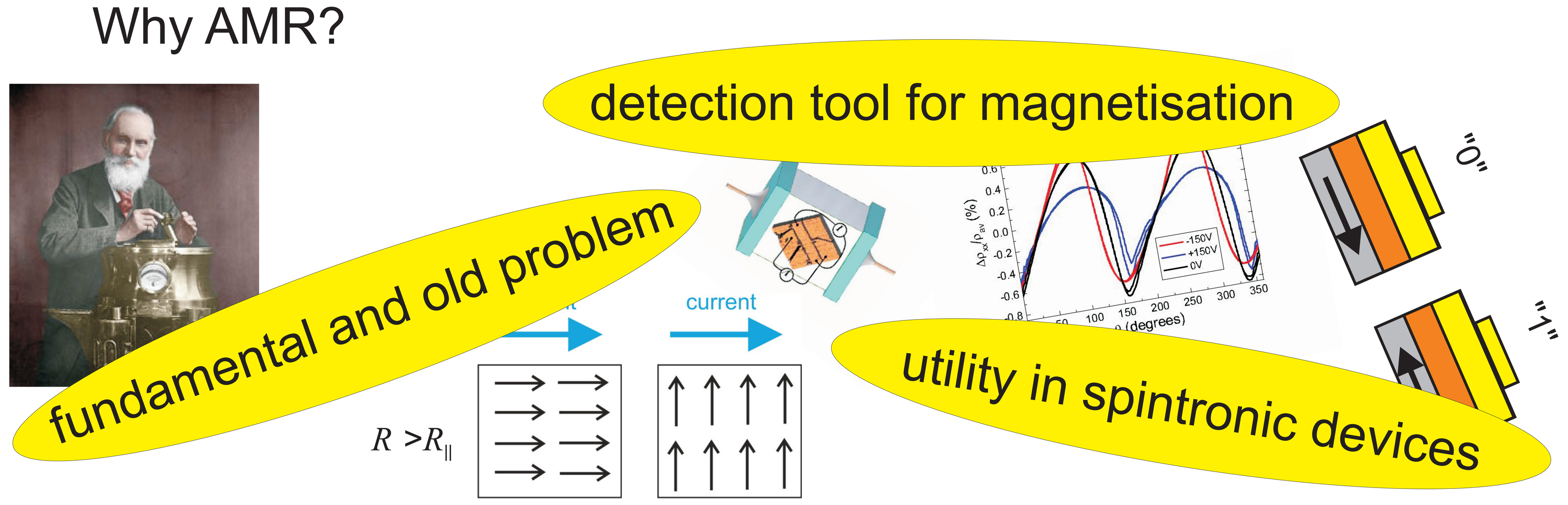
Origin of the non-crystalline Anisotropic MagnetoResistance in (Ga,Mn)As

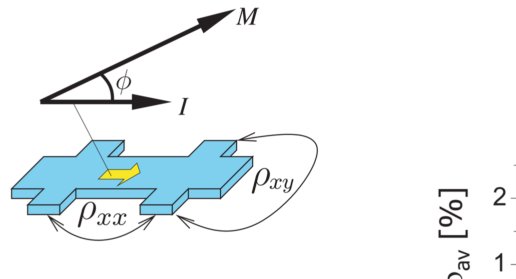
Why AMR?



Phenomenology

Isotropic systems

$$\rho_{xx} = \rho_{\perp} + (\rho_{\parallel} - \rho_{\perp})\cos^2\phi$$
$$\rho_{yy} = (\rho_{\parallel} - \rho_{\perp})\cos\phi\sin\phi$$

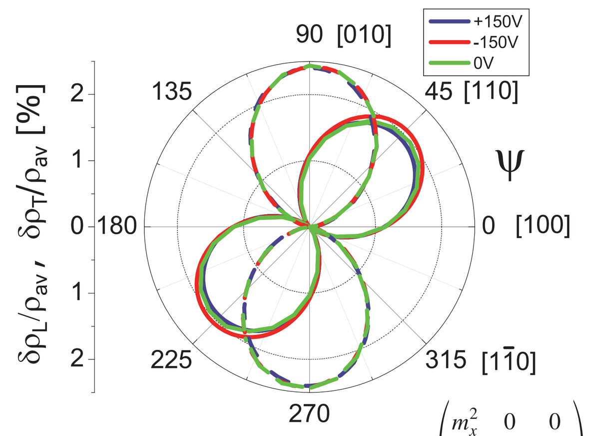


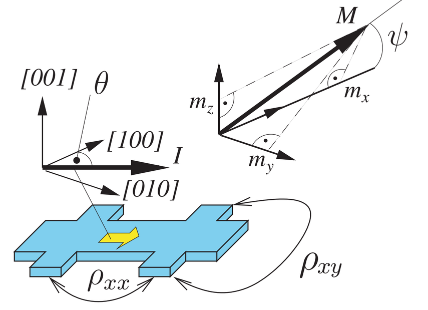
$$\mathrm{AMR} = -2\frac{\sigma_{\parallel} - \sigma_{\perp}}{\sigma_{\parallel} + \sigma_{\perp}} = 2\frac{\rho_{\parallel} - \rho_{\perp}}{\rho_{\parallel} + \rho_{\perp}}$$

Crystals - 2D case (in-plane geometry)

$$\frac{\rho_L - \rho_{av}}{\rho_{av}} = C_I\cos 2(\psi - \theta) + C_{IC}\cos 2(\psi + \theta) + C_C\cos 4\psi + \ldots$$
$$\frac{\rho_T}{\rho_{av}} = C_I\cos 2(\psi - \theta) - C_{IC}\cos 2(\psi + \theta) + \ldots$$

Experimental



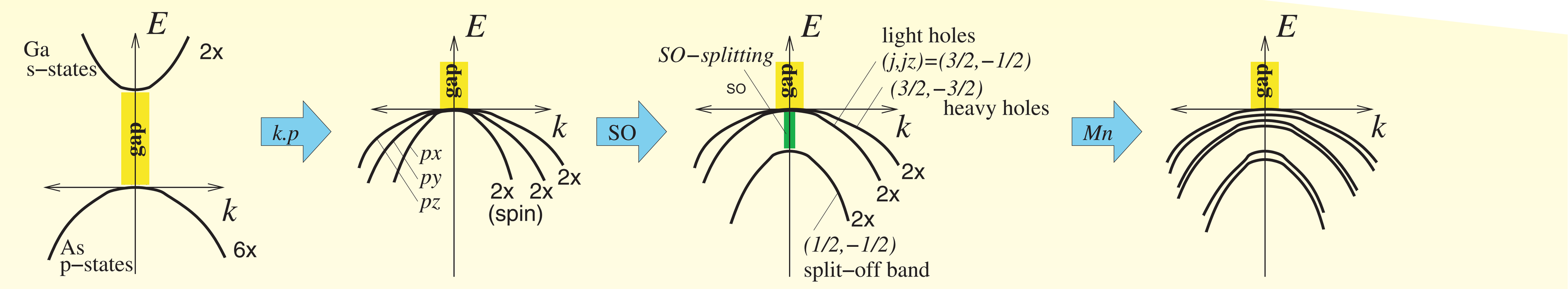


3D case - Limmer et al. [3]:

$$\bar{\rho}_{\text{cubic}} = A\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix} + B\begin{pmatrix}0&m_z&-m_y\\-m_z&0&m_x\\m_y&-m_x&0\end{pmatrix} + C_1\begin{pmatrix}m_x^2&0&0\\0&m_y^2&0\\0&0&m_z^2\end{pmatrix} + C_2\begin{pmatrix}0&m_xm_y&m_xm_z\\m_xm_y&0&m_ym_z\\m_xm_z&m_ym_z&0\end{pmatrix} + D\begin{pmatrix}0&m_z^3&-m_y^3\\-m_z^3&0&m_x^3\\m_y^3&-m_x^3&0\end{pmatrix} +$$
$$+ E_1\begin{pmatrix}m_x^4&0&0\\0&m_y^4&0\\0&0&m_z^4\end{pmatrix} + E_2\begin{pmatrix}m_y^2m_z^2&0&0\\0&m_x^2m_z^2&0\\0&0&m_x^2m_y^2\end{pmatrix} + E_3\begin{pmatrix}0&m_xm_ym_z^2&m_xm_y^2m_z\\m_xm_ym_z^2&0&m_x^2m_ym_z\\m_xm_y^2m_z&m_x^2m_ym_z&0\end{pmatrix},$$

Model definition

1) Band structure (VCA+MF)



- 6x6 Kohn-Luttinger Hamiltonian of GaAs valence band
- kinetic exchange with Mn d-states
- Virtual Crystal Approximation/Mean Field

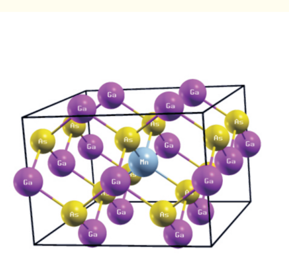
$$H = H_{KL} + V_{dis} = H_{KL} + J_{pd}\sum_I \vec{S}_I\cdot\vec{s}\,\delta(\vec{r} - \vec{R}_I) + \sum_I V(\vec{r} - \vec{R}_I)$$

$$H = H_{KL} + h\hat{e}_M\cdot\vec{s}$$

two non-commuting operators! - Zeeman & SO

2) Scattering

- correction beyond VCA - necessary for finite resistance
- scattering rates by Fermi golden rule
- only substitutional Mn_{Ga} considered
- coherent sum of electric and magnetic scattering potential

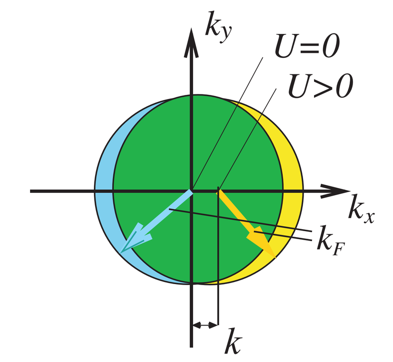


$$\Gamma_{n,\vec{k}} = \frac{2\pi}{\hbar}N_{\rm Mn}\sum_{n'}\int\frac{d^3k'}{(2\pi)^3}|M_{nn'}^{\vec{k}\vec{k}'}|^2\delta(E_n(\vec{k}) - E_{n'}(\vec{k}'))(1 - \cos\theta_{vv'}),\qquad M_{nn'}^{\vec{k}\vec{k}'} = \langle z_{\vec{k}'n'}|M^B + M^C|z_{\vec{k}n}\rangle$$

magnetic Mn moments $M^B = J_{pd}S_{\rm Mn}\hat{e}_M\cdot\vec{s}$

$$M^C = V(|\vec{k} - \vec{k}'|),\quad V(q) = -\frac{e^2}{\varepsilon}\frac{1}{q^2 + q_{TF}^2}\qquad q_{TF} = \sqrt{e^2g/\varepsilon}\propto\sqrt{k_F}$$

screened Coulomb potential (Mn = ionized acceptor)

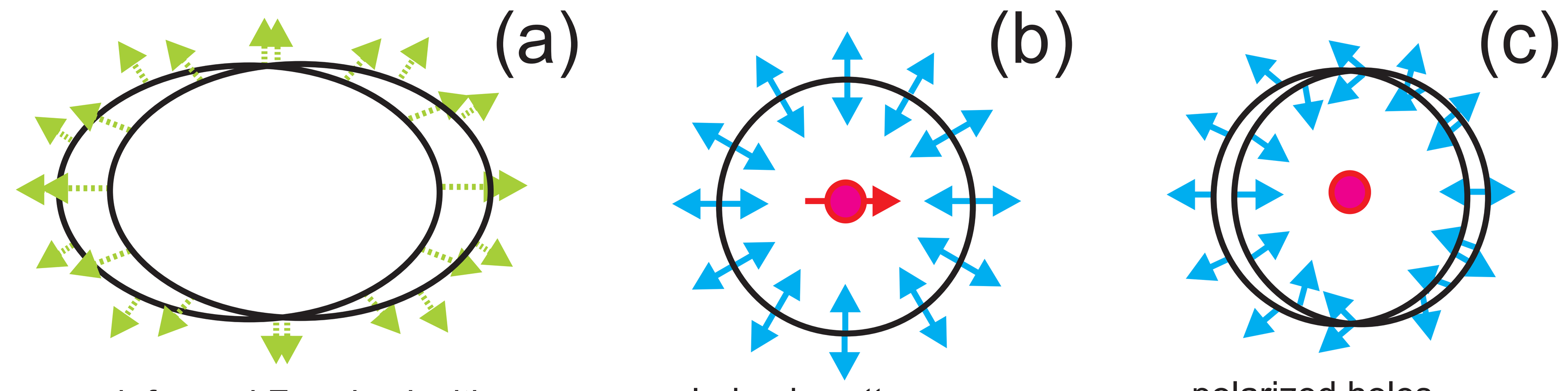


Fermi distribution under influence of electric field (voltage)

3) Transport formalism: Boltzmann equation

$$\sigma_{ij} = e^2\sum_n\int\frac{d^3k}{(2\pi)^3}(\hbar\Gamma_{n\vec{k}})^{-1}v_n^i(\vec{k})v_n^j(\vec{k})\delta(E_F - E(\vec{k})).$$

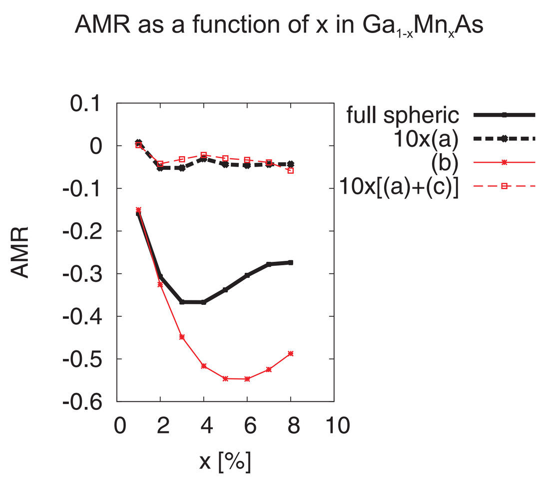
Possible mechanisms of the non-crystalline AMR



deformed Fermi velocities — polarized scatterers — polarized holes

anisotropic band structure — anisotropic relaxation times

... their importance estimated numerically in the model



x enters the model via

- hole density
- exchange field
- (concentration of scatterers)

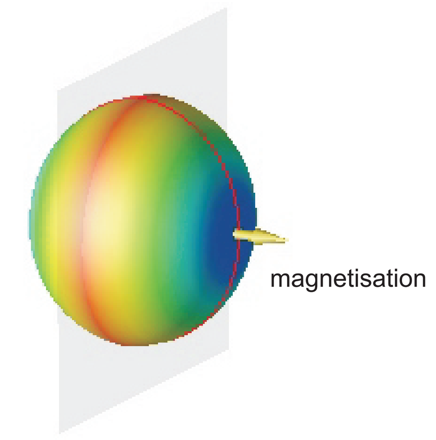
AMR almost vanishes by

- putting (k)=const,
- setting J_{pd}=0 during the evaluation of (k)

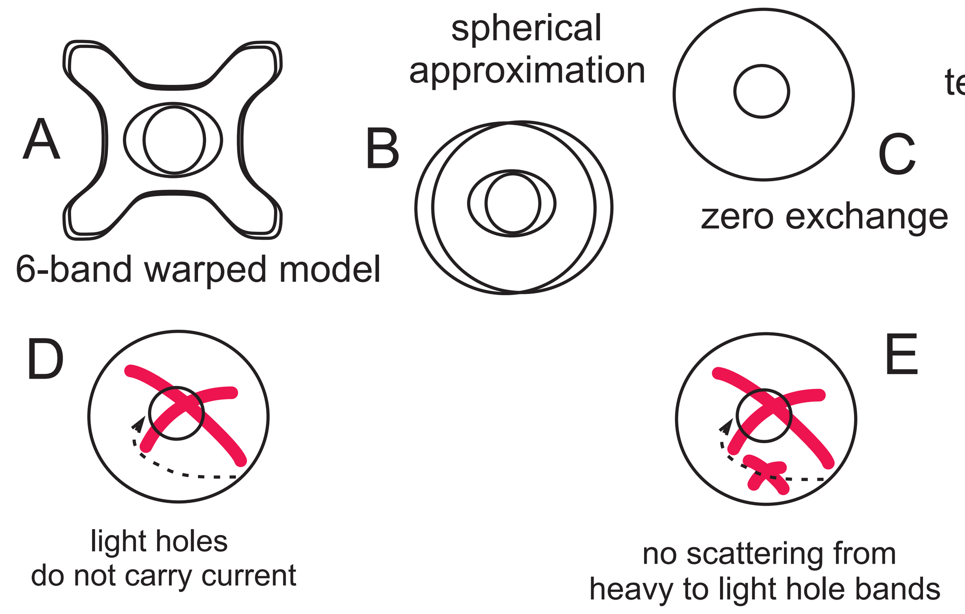
Conclusion:

- scatterer anisotropy [mech. (b)] crucial
- wavefunction anisotropy [mech. (c)] causes quantitative corrections
- mechanisms are not always additive: (b)+(c) (b+c)

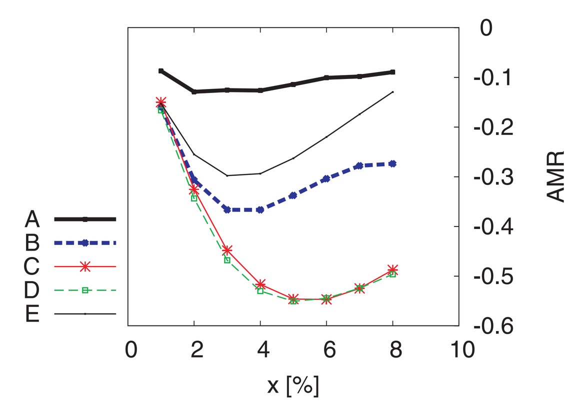
Minority heavy-hole Fermi surface, colour-coded scattering rates



Anisotropic scattering time: simplifying the model



terminus: two spherical heavy hole bands



Outlook - crystalline AMR

Framework: phenomenological formulae

$$\rho_L = \overbrace{(\cos\theta, \sin\theta)}^{\text{direction of the current}}\cdot\hat{\rho}\cdot\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix},\qquad \rho_T = (\cos\theta, \sin\theta)\cdot\hat{\rho}\cdot\begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix}.$$

resistivity: material property (depends only on)

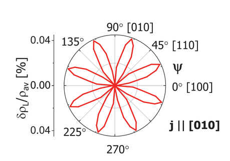
$$\hat{\rho} = \begin{pmatrix}\rho_{11}(\cos\psi, \sin\psi) & \rho_{12}(\cos\psi, \sin\psi)\\ \rho_{21}(\cos\psi, \sin\psi) & \rho_{22}(\cos\psi, \sin\psi)\end{pmatrix},$$

Symmetry constrains the most general form
Example: cubic crystal

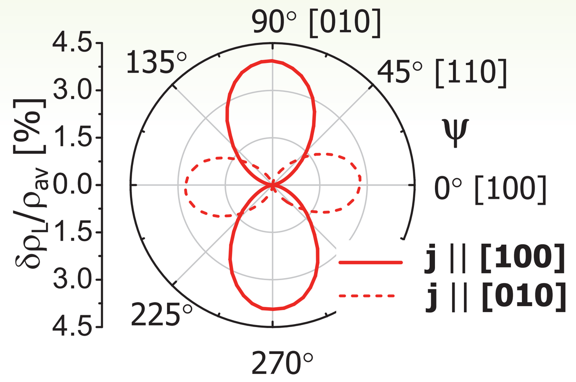
$$\hat{\rho}_{cub} = \begin{pmatrix}u(\cos^2\psi) & \cos\psi\sin\psi\frac{v(\cos^2\psi) + v(\sin^2\psi)}{2}\\ \cos\psi\sin\psi\frac{v(\sin^2\psi) + v(\cos^2\psi)}{2} & u(\sin^2\psi)\end{pmatrix}$$

- functions u and v are arbitrary
- expand into Taylor/Fourier series

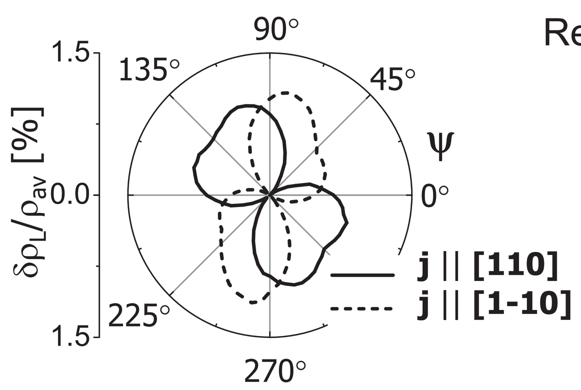
$$\frac{\Delta\rho_L}{\rho_{av}} = C_C\cos 4\psi + C_{C8}\cos 8\psi + \ldots$$
$$+ C_I\cos(2\psi - 2\theta) + C_{IC}\cos(2\psi + 2\theta) +$$
$$+ C_{I6}\cos(6\psi - 2\theta) + C_{IC6}\cos(6\psi + 2\theta) +$$
$$\ldots,$$
$$\frac{\rho_T}{\rho_{av}} = 0 +$$
$$+ C_I\sin(2\psi - 2\theta) - C_{IC}\sin(2\psi + 2\theta) +$$
$$+ C_{I6}\sin(6\psi - 2\theta) - C_{IC6}\sin(6\psi + 2\theta) +$$
$$\ldots$$



Experiments: crystalline AMR
may be enhanced by strain (lithography)

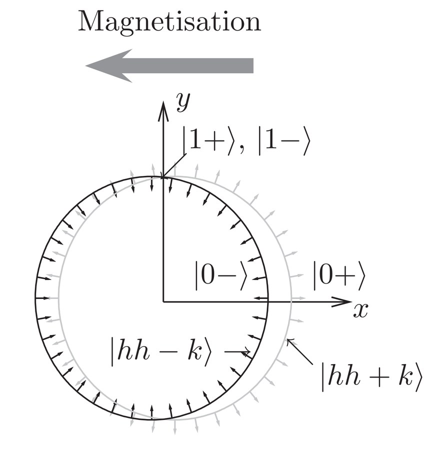


Ref [2]



Analytical model:

$$M^B + M^C = J_{pd}S_{\rm Mn}\hat{e}_M\cdot\vec{s} + V(|\vec{k} - \vec{k}'|) \Rightarrow \alpha + s_x$$



For vanishing exchange, the heavy hole bands are displaced circles, wavefunctions are have s||k

Effective electric scattering strength:

$$\alpha = \frac{\langle V\rangle_{FS}}{J_{pd}S_{\rm Mn}} = \frac{e^2/\varepsilon}{J_{pd}S_{\rm Mn}}\cdot\frac{1}{4k_F^2}\ln\left(1 + \frac{4k_F^2}{q_{TF}^2}\right)$$

Scattering rates (cyllindrically symmetric):

$$\Gamma_{n,\vec{k}} = \Gamma_{\pm}(\phi)\propto\frac{1}{6}\cos^2\phi\pm\alpha\cos\phi + \alpha^2 + \frac{1}{12}$$

Total conductivity parallel to M (|| x):

$$\sigma_{xx} = \sigma_{xx}^{+} + \sigma_{xx}^{-}\qquad \sigma_{xx}^{\pm}\propto\int_{FS}d^2k\cos^2\phi\,\Gamma_{\pm}(\phi).$$

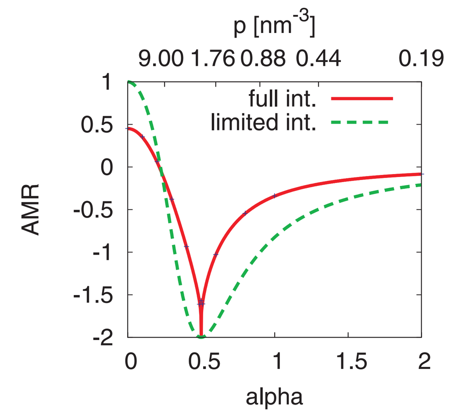
where

$$\sigma_{xx}\propto 12 - 36\alpha\ln\left|\frac{\alpha + \frac{1}{2}}{\alpha - \frac{1}{2}}\right| + \sqrt{18}\frac{24\alpha^2 - 1}{\sqrt{|6\alpha^2 - 1|}}\operatorname{asinh}\frac{\sqrt{|6\alpha^2 - 1|/18}}{|\alpha^2 - \frac{1}{4}|},\quad \alpha^2 > \frac{1}{6}$$

and for ^{2}<1/6 asinh is replaced by asin; further

$$\sigma_{xx}^{\pm} + 2\sigma_{yy}^{\pm} = T^{\pm}\quad\text{and}\quad T^{\pm} = \frac{2\sqrt{18}}{\sqrt{|6\alpha^2 - 1|}}\operatorname{asinh}\frac{\sqrt{|6\alpha^2 - 1|/18}}{|\alpha^2 - \frac{1}{4}|},\qquad \alpha^2 > \frac{1}{6}$$

Simpler - with



Ref [1]

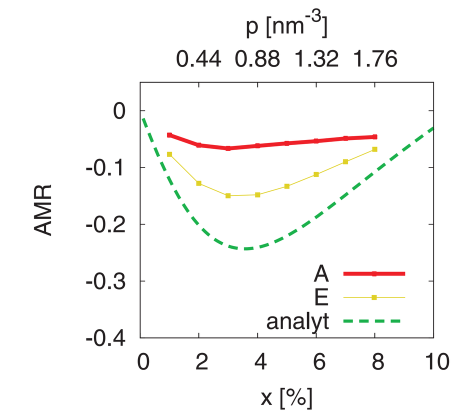
Boltzmann eq. integration: consider only states with wavevector k parallel to the current

$$\sigma_{xx}\propto\frac{1}{\Gamma_+(0)} + \frac{1}{\Gamma_-(0)} + \frac{1}{\Gamma_+(\pi)} + \frac{1}{\Gamma_-(\pi)} = \frac{2}{(\alpha + \frac{1}{2})^2} + \frac{2}{(\alpha - \frac{1}{2})^2}$$
$$\sigma_{yy}\propto\frac{4}{\Gamma_+(\frac{1}{2}\pi)} = \frac{4}{\alpha^2 + \frac{1}{12}},$$

$$\mathrm{AMR} = -\frac{20\alpha^2 - 1}{24\alpha^4 - 2\alpha^2 + 1}$$

- = 0: 'magnetic barrier' - k M feel nothing
- = 1/2: magnetic and electric part cancel each other for k||M (coherent sum!)
- : electric scattering - no anisotropy

More realistic - without



Return to the original scattering model

$$M^B + M^C = J_{pd}S_{\rm Mn}\hat{e}_M\cdot\vec{s} + V(|\vec{k} - \vec{k}'|)$$

and consider again only

$$\Gamma_{\pm}(0) = \frac{\pi}{2}\left[\frac{a^2}{k_F^2}\left(12 + 18b + \frac{8}{b-1} - (1+3b)^2\ln\frac{b+1}{b-1}\right)\mp\right.$$
$$\left.\mp a\left(4 + 6b + 6b^2 - (b+1)(1+3b^2)\ln\frac{b+1}{b-1}\right) + 2k_F^2\right],$$

$$\Gamma_{\pm}(\pi) = \Gamma_{\mp}(0)$$

$$\Gamma_{\pm}(\tfrac{1}{2}\pi) = \Gamma_{\pm}(-\tfrac{1}{2}\pi) = \frac{\pi}{2}\left[\frac{1}{3}k_F^2 + \frac{a^2}{k_F^2}\left(12 + 18b + \frac{8}{b-1} - (1+3b)^2\ln\frac{b+1}{b-1}\right)\right],$$

where

$$a = -(e^2/2\varepsilon k_F^2)/J_{pd}S_{\rm Mn}\qquad b = -(1 + b_0/k_F)\qquad b_0 = q_{TF}^2/k_F^2 = (e^2/\varepsilon)\cdot(2m/h^2k_F)$$

- singularity at a=1/2 is removed
- but overall character of the AMR remains unchanged

Conclusions

- AMR = combined effect of spin-orbit interaction and magnetisation
- general classification of (non-crystalline) AMR sources presented
- non-crystalline AMR in (Ga,Mn)As primarily due to anisotropic scattering times introduced by anisotropic scatterers (Mn magnetic moments); caveat: coherent addition of the electric part (Mn = ionized acceptor)
- suggested experimental test: systems with different effective electric scattering strength

References: [1] Rushforth et al., Phys. Rev. Lett. 99, 147207 (2007); Rushforth et al.,J. Magn. Magn. Mat. (2008), doi:10.1016/j.jmmm.2008.04.070; [2] de Ranieri et al., New J. Phys. 10, 065003 (2008) [3] Limmer et al., Phys. Rev. B 77, 205210 (2008); Limmer et al., Phys. Rev. B 74, 205205 (2006); [4] Jungwirth et al., Appl. Phys. Lett. 81, 4029 (2002); Jungwirth et al.,Appl. Phys. Lett. 83, 320 (2003) [5] Jungwirth et al., Rev. Mod. Phys. 78, 809 (2006) [6] Overby et al., Appl. Phys. Lett. 92, 192501 (2008); Rushforth et al.,arxiv:0801.0886 (to appear in Phys. Rev. B) [7] W. Thomson, Proc. Roy. Soc. (London) 8, 546 (1857)

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