Spin structures in inhomogeneous fractional quantum Hall systems

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Abstract. We investigate the transition from the spin–singlet to the polarized incompressible ground state in systems with filling factor $2/3$ using exact diagonalization with eight electrons. Directly at the transition the ground state is a new stable half–polarized state which seems to be gapless. This state reacts strongly to magnetic inhomogeneities but it prefers stripe–like spin structures to formation of domains.

Fractional quantum Hall systems are known to allow, under some conditions, more incompressible ground states differing by spin polarization. Perhaps the most prominent example [1] are the polarized and the spin–singlet ground states (GS) at filling factor $\nu = 2/3$. Energies of these two GS's may be brought to coincidence by choosing an appropriate electron density (or equivalently magnetic field $B = B_C$ at constant ν) and the system might constitute a model example of a quantum Hall ferromagnet in the fractional regime. Several effects resembling ferromagnetism were measured under these conditions (hysteresis, huge longitudinal magnetoresistance [2], Barkhausen jumps [3]). On the other hand, optical measurements of the spin polarization γ at $\nu = \frac{2}{3}$ and varying B showed signs of a stable transition state: when B approaches B_C there are well pronounced plateaus in $\gamma(B)$ at exactly $\gamma = 0.5$[4]. Deeper theoretical study of the transition in the ground state is thus required.

The standard Yoshioka model[5] replaces the infinite plane by a rectangular primitive cell (of size a by b) with periodic boundary conditions (PBC) in both directions and restricts the one-particle states in it to the lowest Landau level (LLL); the latter step quenches the kinetic energy into an irrelevant constant ($\frac{1}{2}\hbar\omega_c$). For a given number of electrons N_e and filling factor ν this allows only a finite number of N_e–particle states; we will focus on $N_e = 8$ in this work. The choice of the basis is thus the principal approximation whereas the Hamiltonian can be taken in its full complexity. Its matrix is then evaluated and diagonalized.

We considered systems with full Coulomb interaction $H_{\mathrm{Coul}} \propto \sum_{i<j} |r_i - r_j|^{-1}$ and systems with a short–range one H_{hc} (similar to [6]) which enforces the correct type of correlations (like vortices of the wavefuction sitting at the electrons for $\nu = 1/3$) and tends to give results converging faster to the thermodynamical limit. We extended this basic model by including (i) a homogeneous Zeeman term H_{Zeeman} which can bring two states of H_{Coul} with different total S_z to coincidence and (ii) a magnetic inhomogeneity (MI), H_{MI} which (added to H_{Zeeman}) can be interpreted as a Zeeman splitting with spatially varying g–factor. Such a term can then e.g. favourize the sin-

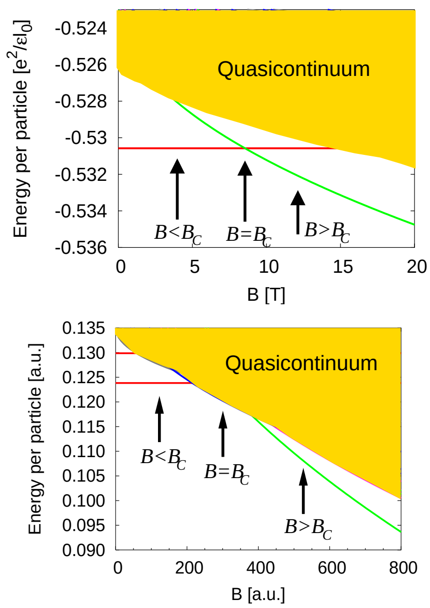

FIGURE 1. Energy versus magnetic field of an eight electron homogeneous system with Zeeman term at constant filling factor $2/3$. Above: Coulomb interaction; direct transition from the singlet ($B < B_C$) incompressible GS to the polarized incompressible GS ($B > B_C$), quasicontinuum is well above the crossing. Below: short–range interaction; a new exactly half–polarized state (on the edge of quasicontinuum) is the GS in the transition region.

glet GS in one part of the system and the polarized GS in another part.

Let us look at a homogeneous system and change the magnetic field while keeping the filling factor constant (i.e. change the electron density and also ratio between Zeeman and Coulomb energy). A direct transition from the incompressible (i.e. gapped) singlet GS to the incompressible polarized GS is observed (Fig. 1 left). This happens owing to the different scaling of H_{Coul} and H_{Zeeman} with B. Now, switch on an inhomogeneity $H_{\mathrm{MI}}(x,y) \propto \cos(x/a)$. This procedure of GS evolution ($S = 0 \to N_e/2$) breaks down[1] already for MI of the order of 10% of the gap (of the singlet GS). Before this breakdown occurs, response of the GS (at the transition point) to MI is weak (polarization varies by at most 10%, while non-correlated electrons are expected to form polarized and unpolarized domains with variation 50%); moreover, even the singlet GS shows then a stronger response [7].

The short–range model also predicts a transition between the two types of ground states (and preserves the nature they have for Coulomb interaction as can be seen from electron–electron correlation functions). However, the transition is not direct (Fig. 1 right): in a finite interval of magnetic fields the absolute GS is another state having $S = N_e/4$ (i.e. half of the full polarization). We would like to stress that this state differs considerably from the polarized and singlet incompressible GSs; it is a quadruplet of states very close to the quasicontinuum and we believe that it will become gapless in thermodynamical limit. This scheme of GS evolution ($S = 0 \to N_e/4 \to N_e/2$) seems not to be bound to hard core systems and is supported by thermodynamical extrapolation of GS energies based on calculations with Coulomb interaction in spherical geometry [8]. Compared to the Coulomb interaction system, this half–polarized ground state (HPGS) sustains stronger MIs (without destroying the scheme $S = 0 \to N_e/4 \to N_e/2$) until nearly full variation of polarization (40%) is reached for MI equal to 25% of the singlet gap (now, the singlet GS is already also affected but less than the HPGS).

In order to study possible spin structures in the HPGS we applied $H_{\mathrm{MI}}(x,y) \propto \delta(x - x_0)$ (plus the PBC). Polarization of the ground state then acquires an $a/2$–periodic form, Fig. 2, which is completely unexpected for an a–periodic $B_z(x)$. This stripe–like structure could be interpreted as a pinned spin density wave. Also slightly stretching the primitive cell of the homogeneous system along x direction induces such structures in the GS (seen in the spin–spin correlation function, Fig. 2).

In conclusion, exact diagonalization studies for eight electrons indicate that no domain formation takes place

[1] For low Zeeman splitting, the GS is no longer an incompressible state but rather lies in the quasicontinuum.

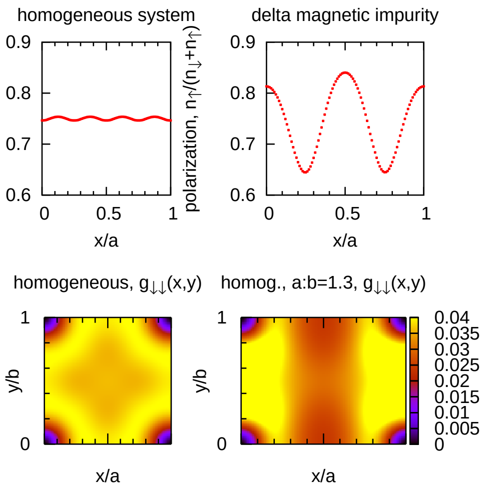

FIGURE 2. Reaction of the half–polarized ($S_z = N_e/4$, $N_e = 8$) ground state to symmetry breaking stimuli. Top: delta magnetic impurity $B_z(x,y) = \delta(x - 0.5a)$ induces two nearly equal maxima along x in polarization. Bottom: stretching the primitive cell along x direction (spin–spin correlation function $g_{\downarrow\downarrow}(r) = \langle \delta(r_1 - r_2 - r)\delta_{\sigma_1\downarrow}\delta_{\sigma_2\downarrow}\rangle$ is shown) makes the electrons align into a stripe in y direction.

at the transition between two incompressible ground states with different spin polarization at $\nu = 2/3$. Instead, a new stable exactly half–polarized state becomes the ground state near the transition point which we expect to be gapless for larger systems. It responds more strongly to magnetic inhomogeneities and it has a tendency to form stripe–like structures when symmetry of the system is disturbed. Further studies of systems with larger number of particles are needed and in process.

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