Magnetoresistance Calculations for a Two-Dimensional Electron Gas with Unilateral Short–Period Strong Modulation

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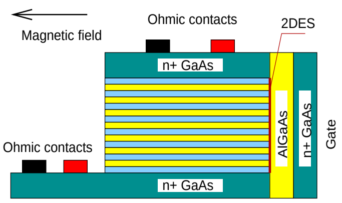
Abstract

Common models describing magnetotransport properties of unilateral superlattices often either directly start from the semiclassical approach or give results well understandable within the semiclassical framework. Recently, oscillations in magnetoresistance have been found on samples with short period ($d = 15$ nm) and strong modulation which cannot be explained on a semiclassical level (magnetic breakdown [1]). We use a simple fully quantum mechanical model which gives us both magnetoresistance data nicely comparing to the experiments and a good intuitive insight into the effects taking place in the system.

The System: seen by experimentators

Experiments by R. A. Deutschmann et al.



- GaAs/GaAlAs superlattice grown between two highly doped n+ GaAs layers
- cleaved in situ perpendicular to the superlattice layers
- AlGaAs spacer followed by a highly doped n+ GaAs layer (gate) grown on the cleaving plane
- positive voltage (with respect to the superlattice) applied to the gate ⇒ triangular potential well at the interface superlattice/GaAlAs spacer ⇒ only its lowest subband occupied ⇒ laterally modulated two dimensional system (2DES)

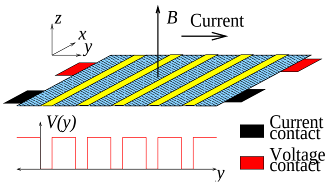
Resistance measured as a function of

- magnetic field
- gate voltage

System Parameters

period of superlattice	15 nm
barrier width	3.1 nm
first modulation band width	3.8 meV
2D concentrations of electrons	$0.5 \div 5.0 \times 10^{-11}$ cm^{-2}

The System: seen by theoreticians



- two dimensional electron gas with unilateral periodic modulation
- subject to perpendicular magnetic field
- concentration of electrons can be manipulated by gate voltage
- concentrations are low, modulation is strong and it has very short period (⇒ only one modulation band is occupied)

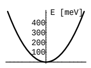
References

[1] R. A. Deutschmann *et al.*, Physica E **6**, 561 (2000).

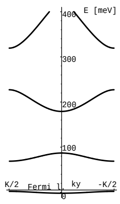
[2] R. A. Deutschmann, Ph.D. thesis, University of Technology Munich, 2001.

[3] K. Výborný, L. Smrčka, and R. A. Deutschmann, cond–mat/0206212 .

2D modulated system at $B = 0$



- free motion along the wires (x direction)



- periodic rectangular potential across the wires (y direction) ⇒ Kronig–Penney model

Lowest band

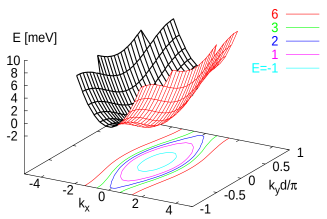
- 3.8 meV wide
- nearly cosine form
- well separated from the second band (60 meV)

Experimentally accessible electron concentrations are low

⇒ Fermi level lies below the second modulation band

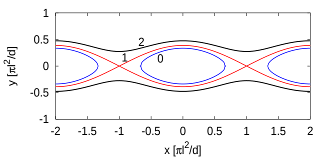
⇒ we consider only the lowest band, i.e. spectrum is

$$E(k_x, k_y) = \frac{\hbar^2}{2m_*}k_x^2 - 2|t|\cos k_y d\,.$$



Magnetotransport: semiclassical approach

1. Determine the trajectory of electron in magnetic field for a given Fermi level E_F
   - Find the cross–section of the *zero field* Fermi surface in (k_x, k_y) space: $E_F = E(k_x, k_y)$.
   - Rotate the (k_x, k_y)–trajectory by $90°$ and scale it by $\hbar/eB$ → real–space trajectory.
2. Quantization
   - open trajectory ⇒ no quantization condition (all such trajectories are allowed).
   - closed trajectory ⇒ the enclosed area must contain an integer multiple of flux quanta $\Phi_0 = h/e$.



In our system therefore:

- for E_F crossing the modulation band: closed trajectories ⇒ quantization $A \cdot \hbar/eB = nh/e, n = 1, 2, \ldots$ (A is the enclosed area in k space) ⇒ Shubnikov–de Haas oscillations (of magnetoresistance) periodic in $1/B$
- for E_F lying above the modulation band: open trajectories ⇒ no magnetoresistance oscillations

Problems:

- $1/B$–periodic oscillations observed experimentally even in the case of open trajectories
- semiclassical model cannot account for 2D→1D transition due to decoupling of wires in strong magnetic field

Quantum mechanical model (in magnetic field)

$$H = H_x + H_y = \frac{1}{2m}(p_x + eBy)^2 + \frac{1}{2m}p_y^2 + V(y)$$

Restriction to the lowest modulation band is equivalent to tight binding ansatz

$$\Psi(x, y) = \frac{1}{\sqrt{2\pi}}\exp(ikx)\sum_j a_j(k)\varphi(y - jd)$$

$$\langle\varphi_i|H_y|\varphi_j\rangle = -|t|\delta_{i,j\pm1}$$

where $\varphi(y - jd)$ or $|\varphi_j\rangle$ denotes the ground state (more precisely Wannier state) in the j-th well of modulation potential. We thus obtain a matrix problem

$$H_{ij}(k) = |t|\left[\alpha^2\left((k/K) + i\right)^2\delta_{i,j} - \delta_{i,j\pm1}\right],$$

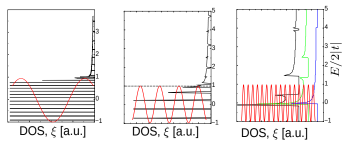
$$\alpha^2 = \frac{e^2B^2}{m}\cdot\frac{d^2}{2|t|} = \left(\frac{\hbar\omega_{\text{eff}}}{2|t|}\right)^2, \qquad K = d\frac{eB}{\hbar}$$

Spectrum

- depends (up to scaling by $|t|$) on a single dimensionless parameter α
- coincides with spectrum of a 1D particle subject to periodic cosine potential (Mathieu problem)

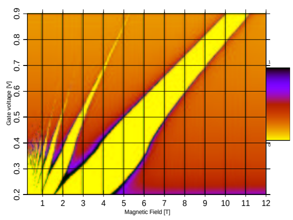
$$-\frac{\hbar^2}{2m}\psi''(\xi) - \psi(\xi)2|t|\cos K\xi = E\psi(\xi)$$

Density of states of fictitious particle in periodic cosine potential



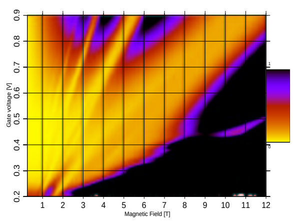
$\alpha \ll 1$	$\alpha \approx 1$	$\alpha > 1$
weak field	intermediate field	strong field

Density of states



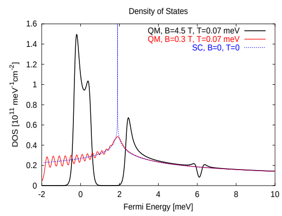
Experimental magnetoresistance

Measured by R. A. Deutschmann [1],[2].



Conclusion: gaps in the density of states match very well to the oscillation extrema in the magnetoresistance.

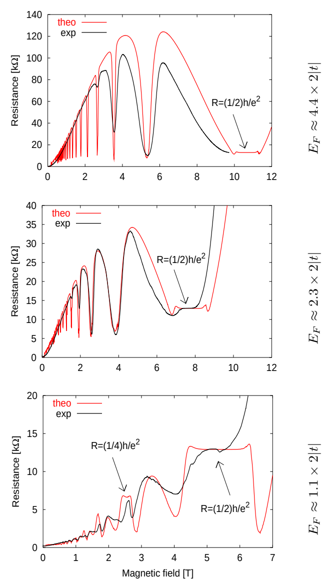
Comparison between quantum–mechanical and semiclassical approach



Low magnetic field	High magnetic field
$\alpha \ll 1$	$\alpha \gtrsim 1$
the $B = 0$ DOS + quantization condition for $E < 2\|t\|$ ≈ the QM DOS	QM DOS and $B = 0$ DOS fundamentally different, esp. gaps in the $E > 2\|t\|$ region
SC approach works	magnetic breakdown
	SC approach fails

Transport calculations

- Kubo formula (linear response to applied bias)
- impurity scattering: isotropic, relaxation time approximation, ansatz $\hbar/\tau = \Gamma \propto \sqrt{B}$ for the imaginary part of self–energy (which may be interpreted as inverse relaxation time)
- measured resistance is $\varrho_{xy} + c\varrho_{yy}$ (due to two–point–contact geometry)



Conclusion [3]

- semiclassical approach fails to predict correct magnetoresistance behaviour for unilateral short period superlattices with strong modulation
- a simple quantum mechanical model based on tight binding approximation can account very well for the $1/B$–periodic oscillations in magnetoresistance
- transport calculations based on this model give results in almost quantitative agreement with the experiments for higher electron concentrations; for lower concentrations, the agreement is worse but still the main features of the experimental data are retained