Spin structures in inhomogeneous fractional quantum Hall systems

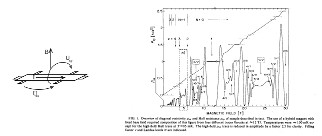
KAREL VÝBORNÝ AND DANIELA PFANNKUCHE
I. Institute of Theoretical Physics, University of Hamburg, Jungiusstr. 9, 20355 Hamburg, Germany

Abstract

Discovery of the huge longitudinal magnetoresistance (HLM) phenomenon [1] opened new promising ways to measure indirectly nuclear spin polarization in GaAs/GaAlAs heterostructures by means of conductivity measurements rather than by e.g. NMR [2]. The HLM has been experimentally studied on high–mobility two–dimensional electron gases in the fractional quantum Hall regime (filling factor $\frac{2}{3}$) where the ground state is known to be spin unpolarized for lower magnetic fields and spin polarized for higher magnetic fields [3]. Although there are strong hints that the phenomenon appears due to formation of domains of spin polarized and spin unpolarized states there is – to our best knowledge – neither theoretical nor direct experimental evidence for this model so far.
We report on finite size calculations based on the standard model developed by Yoshioka et al. [4] with a magnetic inhomogeneity added. We study the spin structures appearing in the ground state near the critical field where spin polarization of the ground state changes in a homogeneous system.

Experiments

Fractional quantum Hall effect
Tsui, Störmer, Gossard 1982
Experimental setup   Willett *et al.*, PRL **59**, 1776 (1987)



Huge longitudinal magnetoresistance (HLM)
Kronmüller *et al.*, PRL **81**, 2526 (1998)

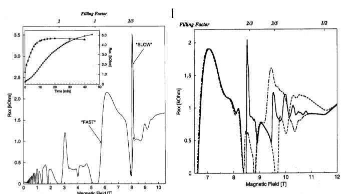


FIG. 1. The longitudinal resistance of a Hall bar at 0.4 K. If the magnetic field is swept upwards very slowly (0.002 T/min, trace "slow"), one observes a very prominent and sharp huge resistance maximum at filling factor $\frac{2}{3}$. The inset shows the temporal evolution of the HLR for two different sample widths [800 μm (dots) and 80 μm (triangles)].

Fig. 2. Longitudinal resistance measurements at 40 mK of a 80 µm wide Hall bar. Dashed trace: up-sweep of the magnetic field (0.3 T/min). Dot-Dashed trace: down-sweep of the magnetic field (0.3 T/min). Solid line: down-sweep at slower sweep rates (0.006 T/min). The HLR is visible in the down-sweep of the magnetic field and not in the up-sweep. But it develops fully only at slower sweep rates (0.006 T/min).

• big resistance peak appears at filling factor $\nu = \frac{2}{3}$ if magnetic field is swept slowly;
• the peak shows hysteretic behaviour
• it is known that the ground state at $\nu = \frac{2}{3}$ can be either spin unpolarized ($\updownarrow$) or fully spin polarized ($\uparrow\uparrow$) depending on the Zeeman splitting [3]
$\Rightarrow$ possible explanation of HLM: domains of $\updownarrow$ and $\uparrow\uparrow$ states form due to inhomogeneities; domain walls enhance sample resistance
• inhomogeneity may be made up of polarized (domains of) nuclear spins which have macroscopic relaxation times

References

[1] S. Kronmüller, W. Dietsche, J. Weis, K. von Klitzing, W. Wegscheider, and M. Bichler. New resistance maxima in the fractional quantum hall effect regime. *Phys. Rev. Lett.*, 81(12):2526–2529, 1998.

[2] J.H. Smet, R.A. Deutschmann, F. Ertl, W. Wegscheider, H. Abstreiter, and K. von Klitzing. Gate–voltage control of spin interactions between electrons and nuclei in a semiconductor. *Nature*, 415:281–286, 2002.

[3] T. Chakraborty. Spin–reversed ground state and energy gap in the fractional quantum hall effect. *Surf. Sci.*, 229:16–20, 1990.

[4] D. Yoshioka. Ground state of the two–dimensional charged particles in a strong magnetic field and the fractional quantum hall effect. *Phys. Rev. B.*, 29(12):6833–6839, 1984.

Model (homogeneous systems)

- homogeneous infinite 2D electron gas
- strong perpendicular magnetic field B
- electrons interact via Coulomb interaction

$$\mathbf{H} = \mathbf{H}_{\text{Coul}} = \frac{1}{4\pi\varepsilon\ell}\sum_{i<j}\frac{1}{|\underbrace{\vec{\mathbf{r}}_i/\ell}_{\vec{\rho}_i} - \underbrace{\vec{\mathbf{r}}_j/\ell}_{\vec{\rho}_j}|} + \frac{1}{2m^*}\sum_i(\vec{p} - e\vec{A})^2$$

Magnetic length: $\ell^2 = \hbar/(eB)$

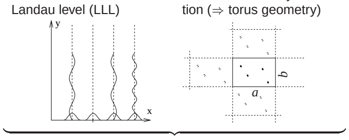
How to treat it

- choose a finite basis ($\equiv$ approximation)
- calculate matrix elements of $\mathbf{H}$ in this basis
- diagonalize ($\rightarrow$ spectra, eigenstates)

$$\mathbf{H} = \frac{1}{4\pi\varepsilon\ell}\sum_{\substack{i_1,i_2,\\ i_3,i_4}}\sum_{\sigma_1,\sigma_2\in\{\uparrow,\downarrow\}} \mathbf{c}^+_{i_1\sigma_1}\mathbf{c}^+_{i_2\sigma_2}\mathbf{c}_{i_3\sigma_2}\mathbf{c}_{i_4\sigma_1}\langle i_1 i_2|\frac{1}{|\vec{\rho}' - \vec{\rho}|}|i_3 i_4\rangle + \hbar\omega\sum_i\left(n_i + \frac{1}{2}\right)\mathbf{c}^+_i\mathbf{c}_i\,.$$

Choosing the basis = approximation

Restriction to the lowest Landau level (LLL)   Periodic boundary condition ($\Rightarrow$ torus geometry)

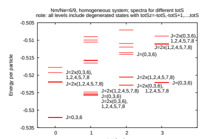


Only m allowed one–electron wavefunctions

$$(j = 0,1,\dots m-1);\; j = \begin{cases}\text{momentum } (||y)\\ \text{position } (||x)\end{cases}$$

$$\left.\begin{array}{l} m = \#\text{ of flux quanta in the rectangle}\\ n = \#\text{ of particles in the rectangle}\end{array}\right\}\text{filling factor: } \frac{n}{m} = \nu$$

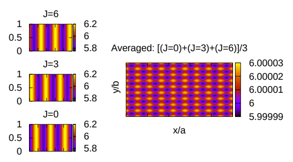
Homogeneous $n/m = 6/9$ system



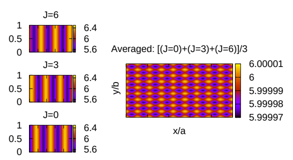
- The absolute ground state occurs at $S = 0$.
- Ground states at $S = 0$ and $S = 3$ have the same structure.

One–particle densities

Ne/Nm=6/9, homogeneous, totS=0; particle densities

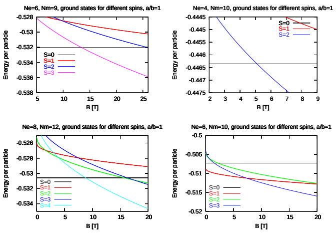


Ne/Nm=6/9, homogeneous, totS=3; particle densities



Homogeneous system + Zeeman term

$$\mathbf{H} = \mathbf{H}_{\text{Coul}} + \underbrace{\left(-\mu_B g B_z\right)\sum_{i=1}^{n}\mathbf{s}^i_z}_{\mathbf{H}_{\text{Zeeman}}}$$



- For $n/m = 6/9$ and $n/m = 8/12$ there is a transition from a spin unpolarized ($S = 0, S_z = 0$) state to a fully spin polarized state ($S = 3, S_z = 3$),($S = 4, S_z = 4$).
- Spin polarization changes abruptly at the transition (crossing).

Inhomogeneities

Weak fluctuating magnetic field perpendicular to the 2D gas ($\mathbf{H}_{\text{MI}}$) and $||\ x$ ($\mathbf{H}_{\text{IMI}}$).

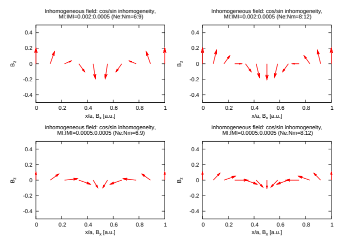
$$\mathbf{H} = \mathbf{H}_{\text{Coul}} + \mathbf{H}_{\text{Zeeman}} + \underbrace{\mu_B g\sum_{i=1}^{n}B_z(\mathbf{x}_i)\mathbf{s}^i_z}_{\mathbf{H}_{\text{MI}}} + \underbrace{\mu_B g\sum_{i=1}^{n}B_x(\mathbf{x}_i)\mathbf{s}^i_x}_{\mathbf{H}_{\text{IMI}}}$$

Consider $B_z(x) \propto \cos(2\pi x/a)$ and B at the ground state transition.
– $\mathbf{H}_{\text{MI}}$ favours

- the polarized states near to $x = 0$ and $x = a$ ($B \uparrow\uparrow B_z$)
- the unpolarized states near to $x = a/2$ ($B \uparrow\downarrow B_z$)

– $\mathbf{H}_{\text{IMI}}$: necessary to make the crossing states mix

Spatial form of inhomogeneities



Quantities to evaluate

- Particle density: $\mathbf{n}(x) = \sum_{i=1}^{n}\delta(\mathbf{x}_i - x)$.
- Density of total spin z–component (spinZ density): $\mathbf{n}_{S_z}(x) = (\mathbf{n}(x)\otimes\mathbf{S}_z)/\mathbf{n}(x)$. For example at $n = 2$, $S_z = -1,0,1$:

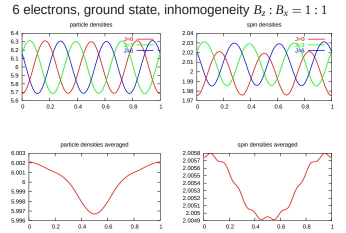
$$\langle\psi|\mathbf{n}(x)\otimes\mathbf{S}_z|\psi\rangle = 1\cdot\langle\psi_1|\mathbf{n}(x)|\psi_1\rangle + 0\cdot\langle\psi_0|\mathbf{n}(x)|\psi_0\rangle - 1\cdot\langle\psi_{-1}|\mathbf{n}(x)|\psi_{-1}\rangle\,.$$

Where $|\psi_1\rangle$ is a projection of $|\psi\rangle$ to the subspace of all states with $S_z = 1$.

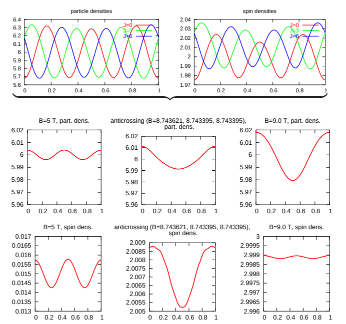
- Density of total spin x–component
- Density of total spin (spin density): $\mathbf{n}_\mathbf{S}(x) = (\mathbf{n}(x)\otimes\mathbf{S}^2)/\mathbf{n}(x)$.
- Density of particles with spin down (spinDown density; 'polarization'): $\mathbf{n}_\downarrow(x) = (\mathbf{n}(x)\otimes\mathbf{P}_\downarrow)/\mathbf{n}(x)$.

Is it a domain?

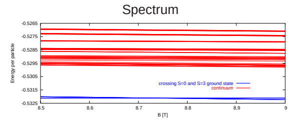
6 electrons, ground state, inhomogeneity $B_z : B_x = 1:1$



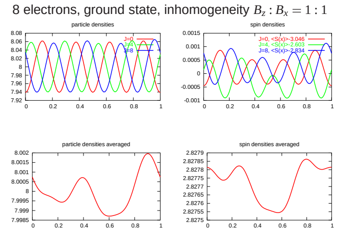
6 electrons, ground state, inhomogeneity: $B_z : B_x = 4:1$



Spectrum



8 electrons, ground state, inhomogeneity $B_z : B_x = 1:1$



Summary

- Owing to the Zeeman splitting there is a transition from a $S = 0$ state to a fully spin polarized state ($S = n/2$) at $B = B_{\text{crit}}$ in the ground state of $\nu = \frac{2}{3}$ homogeneous systems.
- A model inhomogeneity was proposed which makes the crossing states mix $\Rightarrow$ $\langle S\rangle$ of the ground state varies smoothly from 0 to $n/2$ as B passes through B_{crit}.
- Finite size effects: suppressed by averaging the densities over the three nearly degenerated lowest lying states
- Local expectation values of $\langle S\rangle$:
  - mimic the underlying inhomogeneous magnetic field
  - amplitude proportional* to inhomogeneity strength
  - amplitude weak ($\Delta\langle S\rangle/\langle S\rangle \lesssim 0.001$) for inhomogeneities which do not drastically change the spectrum
  - no convincing tendence seen when comparing 6-electron and 8-electron systems
- The studied systems seem to be too small for domains to build up.