Spin structures in inhomogeneous fractional quantum Hall systems

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Motivation

- huge longitudinal magnetoresistance (HLM) phenomenon [1]
  - ⇒ occurs in the $\nu = 2/3$ FQHE state
  - ⇒ allows to measure indirectly nuclear spin polarization in GaAs/GaAlAs heterostructures [2]
- the HLM is not theoretically fully understood

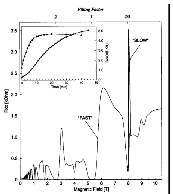


FIG. 1. The longitudinal resistance of a Hall bar at 0.4 K. If the magnetic field is swept upwards very slowly (0.002 T/min, trace "slow"), one observes a very prominent and sharp huge resistance maximum at filling factor 2/3. The inset shows the temporal evolution of the HLR for two different sample widths [800 μm (dots) and 80 μm (triangles)].

Suggested explanation:

- two concurrent correlated ground states: unpolarized and fully polarized [4]
- domains are built up
- scattering on domain walls ⇒ increased resistance

Model (homogeneous systems)

- homogeneous infinite 2D electron gas
- strong perpendicular magnetic field B
- electrons interact via Coulomb interaction ($\ell^2 = \hbar/(eB)$)

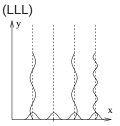
$$\mathbf{H} = \mathbf{H}_{kin} + \mathbf{H}_{Coul} = \frac{e^2}{4\pi\varepsilon\ell}\sum_{i<j}\frac{1}{|\underbrace{\vec{\mathbf{r}}_i/\ell}_{\vec{\rho}_i} - \underbrace{\vec{\mathbf{r}}_j/\ell}_{\vec{\rho}_j}|} + \frac{1}{2m^*}\sum_i(\vec{p} - e\vec{A})^2$$

How to treat it (Yoshioka *et al.* [5])

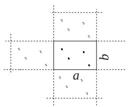
- choose a finite basis (≡ approximation)
- calculate matrix elements of $\mathbf{H}$ in this basis
- diagonalize (→ spectra, eigenstates)

Choosing the basis = approximation

Restriction to the lowest Landau level (LLL)



Periodic boundary condition (⇒ torus geometry)

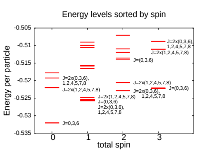


Only m allowed one–electron wavefunctions

$$(j = 0, 1, \ldots m-1);\ j = \begin{cases} \text{momentum } (\|y) \\ \text{position } (\|x) \end{cases}$$

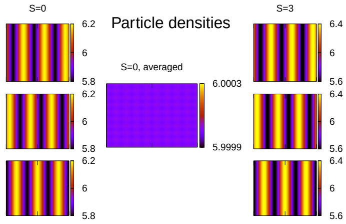
m = # of flux quanta in the rectangle
n = # of particles in the rectangle
} filling factor: $\frac{n}{m} = \nu$

Homogeneous $n/m = 6/9$ system



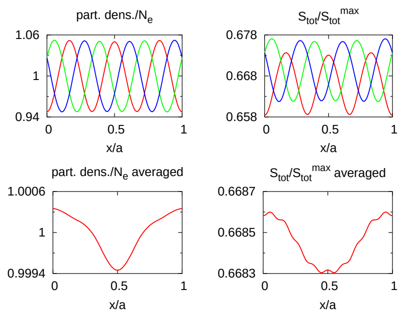
- The absolute ground state occurs at $S = 0$.
- Ground states at $S = 0$ and $S = 3$ have the same structure.

Particle densities

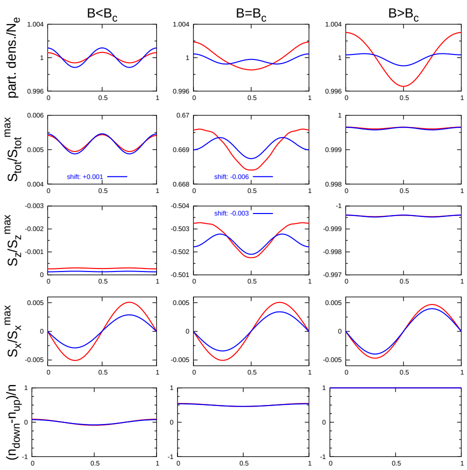


- to filter out finite size effects: average triply (nearly) degenerate ground state

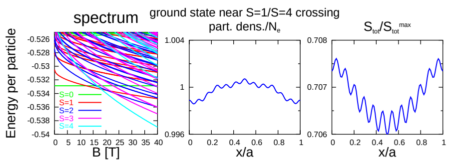
6 electrons, ground state, $\langle \mathbf{H}_{MI} \rangle = 0.0005$ p.p., $\langle \mathbf{H}_{IMI} \rangle = 0.0005$ p.p.



ground state, $\langle \mathbf{H}_{MI} \rangle = 0.002$ p.p., $\langle \mathbf{H}_{IMI} \rangle = 0.0005$, 6 electrons, 8 electrons



8 electrons, aspect ratio 2:1, ground state, $\langle \mathbf{H}_{MI} \rangle = 0.0005$ p.p., $\langle \mathbf{H}_{IMI} \rangle = 0.0005$ p.p.

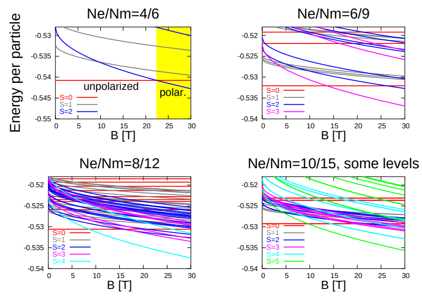


Summary

- A model inhomogeneity was proposed which makes the crossing states mix ⇒ $\langle S \rangle$ of the ground state varies smoothly from 0 to $n/2$ as B passes through B_{crit}.
- Finite size effects: suppressed by averaging the densities over the three nearly degenerated lowest lying states
- Local expectation values of $\langle S \rangle$ and $\langle S_z \rangle$: response to inhomogeneity
  - mimics the underlying inhomogeneous magnetic field
  - proportional to inhomogeneity strength
  - weak ($\Delta\langle S \rangle / \langle S \rangle \lesssim 0.001$) for inhomogeneities which do not drastically change the spectrum
  - weaker in systems with 8 particles compared to system with 6 particles
  - stronger when the aspect ratio of primitive cell is 2:1
- Local expectation values of $\langle S_x \rangle$
  - response to inhomogeneity much stronger ($\chi_x/\chi_z \approx 40$ at crossing)
  - do not react to the ground state change
- Domain state cannot be build solely with the $S = 0$ and $S = N_e/2$ incompressible state.

Homogeneous system + Zeeman term

$$\mathbf{H} = \mathbf{H}_{Coul} + \underbrace{(-\mu_B g B_z)\sum_{i=1}^n s_z^i}_{\mathbf{H}_{Zeeman}}$$



- There is a transition from a spin unpolarized ($S = 0, S_z = 0$) state to a fully spin polarized state ($S = N_e/2, S_z = N_e/2$).
- Spin polarization changes abruptly at the transition (crossing).
- finite gap at the crossing

Inhomogeneities

Weak fluctuating magnetic field perpendicular to the 2D gas ($\mathbf{H}_{MI}$) and $\| x$ ($\mathbf{H}_{IMI}$).

$$\mathbf{H} = \mathbf{H}_{Coul} + \mathbf{H}_{Zeeman} + \underbrace{\mu_B g \sum_{i=1}^n B_z(\mathbf{x}_i) s_z^i}_{\mathbf{H}_{MI}} + \underbrace{\mu_B g \sum_{i=1}^n B_x(\mathbf{x}_i) s_x^i}_{\mathbf{H}_{IMI}}$$

Consider $B_z(x) \propto \cos(2\pi x/a)$ and B at the ground state transition.
– $\mathbf{H}_{MI}$ favours

- the polarized states near to $x = 0$ and $x = a$ ($B \uparrow\uparrow B_z$)
- the unpolarized states near to $x = a/2$ ($B \uparrow\downarrow B_z$)

– $\mathbf{H}_{IMI}$: necessary to make the crossing states mix

Quantities to evaluate

- Particle density: $\mathbf{n}(x) = \sum_{i=1}^n \delta(\mathbf{x}_i - x)$.
- Density of total spin z–component (spinZ density): $\mathbf{n}_{S_z}(x) = (\mathbf{n}(x) \otimes \mathbf{S}_z)/\mathbf{n}(x)$. For example at $n = 2$, $S_z = -1, 0, 1$:

$$\langle\psi|\mathbf{n}(x)\otimes\mathbf{S}_z|\psi\rangle = 1\cdot\langle\psi_1|\mathbf{n}(x)|\psi_1\rangle + 0\cdot\langle\psi_0|\mathbf{n}(x)|\psi_0\rangle - 1\cdot\langle\psi_{-1}|\mathbf{n}(x)|\psi_{-1}\rangle\,.$$

  Where $|\psi_1\rangle$ is a projection of $|\psi\rangle$ to the subspace of all states with $S_z = 1$.
- Density of total spin x–component
- Density of total spin (spin density): $\mathbf{n}_\mathbf{S}(x) = (\mathbf{n}(x) \otimes \mathbf{S}^2)/\mathbf{n}(x)$.
- Density of particles with spin down (spinDown density; 'polarization'): $\mathbf{n}_\downarrow(x) = (\mathbf{n}(x) \otimes \mathbf{P}_\downarrow)/\mathbf{n}(x)$.

References

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[2] J.H. Smet, R.A. Deutschmann, F. Ertl, W. Wegscheider, H. Abstreiter, and K. von Klitzing. *Nature*, 415:281–286, 2002.

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[4] T. Chakraborty. *Surf. Sci.*, 229:16–20, 1990.

[5] D. Yoshioka. *Phys. Rev. B.*, 29(12):6833–6839, 1984.