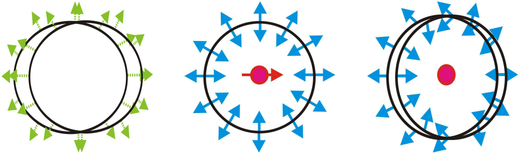
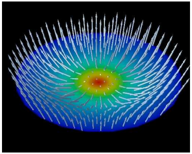
Thin metallic conductor subject to magnetic field:

once in-plane



Anisotropic magnetoresistance in (Ga,Mn)As

once out-of-plane



Skyrmions
in Fractional
quantum Hall effect

Karel Výborný

Tomáš Jungwirth

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Texas A&M University

Alexey A. Kovalev, Jairo Sinova

HITACHI Cambridge

Elisa de Ranieri, Jörg Wunderlich

University of Nottingham

Andrew W. Rushforth, Bryan L. Gallagher, Richard P. Campion

Universität Hamburg

Christian Müller, Daniela Pfannkuche

Leibniz Universität Hannover

Annelene F. Dethlefsen, Rolf J. Haug

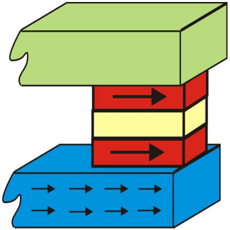
Politechnika Wrocławska

Arek Wójs

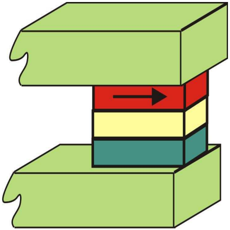
Magnetic field (magnetisation) in-plane

Anisotropic MagnetoResistance and similar...

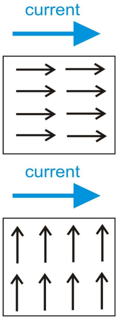
AMR $\neq$ magnetoresistance as $R(|\vec{B}|)$, rather $R(\vec{B}/|\vec{B}|)$



TMR
Fert/Grünberg (1988)



TAMR
Gould et al. (2003)



AMR
W. Thomson (1857)

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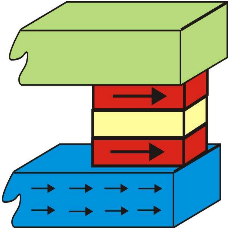
AMR $\neq$ magnetoresistance as $R(|\vec{B}|)$, rather $R(\vec{B}/|\vec{B}|)$



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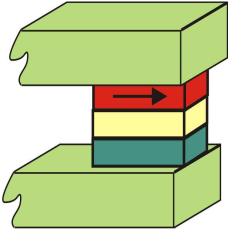

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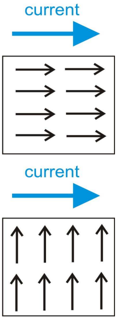


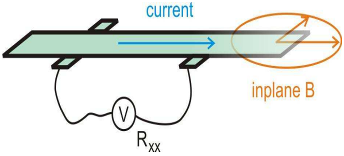
AMR
W. Thomson (1857)

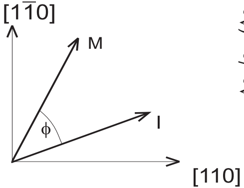
Effect due to
spin-orbit interaction

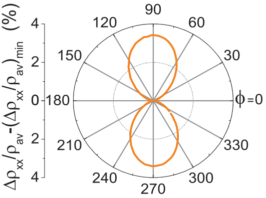
AMR in (Ga,Mn)As thin films

$$\Delta\rho_{xx}/\rho_{av} \quad = \quad C_I \cos 2\phi$$

$$\Delta\rho_{xy}/\rho_{av} \quad = \quad C_I \sin 2\phi$$





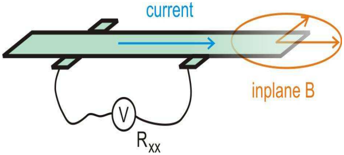


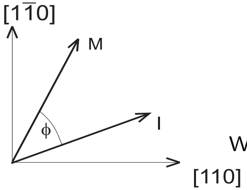
AMR in (Ga,Mn)As thin films

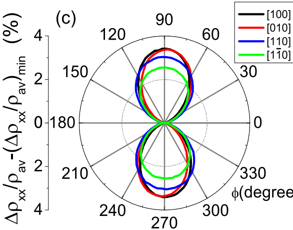
$$\Delta\rho_{xx}/\rho_{av} = C_I \cos 2\phi + C_{I,C}\cos(2\psi + 2\theta) + C_C \cos 4\psi$$

$$\Delta\rho_{xy}/\rho_{av} = C_I \sin 2\phi - C_{I,C}\sin(2\phi + 4\theta)$$

de Ranieri et al., New J. Phys. '08







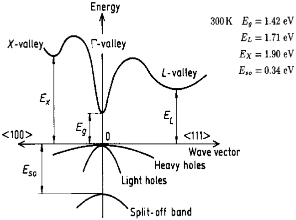
Wunderlich et al., Phys. Rev. B

Magnetic semiconductor $Ga_{1-x}Mn_xAs$: model

6×6 description of the GaAs valence band

$$H = H_{KL}$$

H_{KL} parametrized by $\gamma_1, \gamma_2, \gamma_3$, Δ_{SO}



Magnetic semiconductor $Ga_{1-x}Mn_xAs$: model

6×6 description of the GaAs valence band + kinetic *p*-*d* exchange of As *p*-states with Mn *d*-states

$$H = H_{KL} + J_{pd} \sum_{i,I} S_I \cdot s_i \delta(r_i - R_I)$$

H_{KL} parametrized by $\gamma_1, \gamma_2, \gamma_3$, Δ_{SO}

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mean field, then forget the Mn states

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Scattering — via Fermi golden rule

- mag+non-mag scattering on Mn substituting Ga
- non-mag scattering on interstitial Mn – facultative

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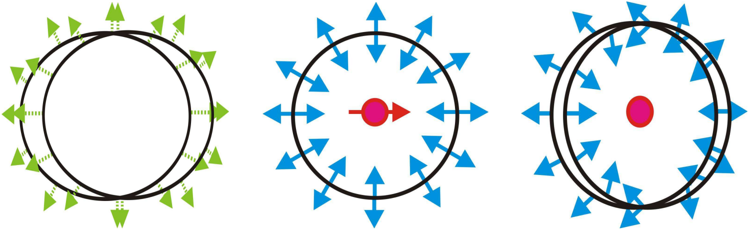
Transport formalism — semiclassical Boltzmann equation

Jungwirth et al., Appl. Phys. Lett. '02; Jungwirth et al., Rev. Mod. Phys. '06

Sources of *non-crystalline* anisotropies

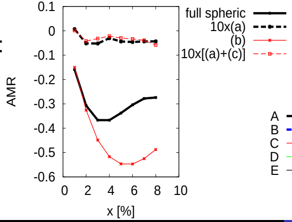
bent bandstructure | wavefunctions

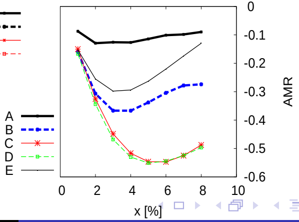


(a) (b) (c)

Fermi velocities | scattering rates

(Ga,Mn)As, with only Mn_{Ga}: mechanism (b) dominant



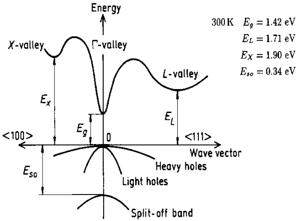


simplifications:
A = warped FS
B = spherical
C = exch.$\rightarrow$ 0
E = h.h. only

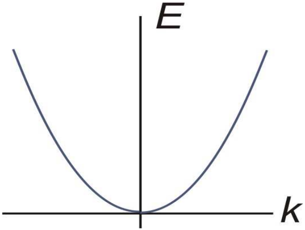
Perpendicular magnetic field

(a part of...) The essence of quantum Hall physics

bandstructure in a solid, e.g. in GaAs:



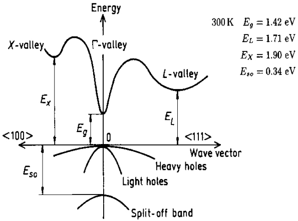
free electrons (in 2D)



$E = \hbar^2 k^2 / 2m^*$

(a part of...) The essence of quantum Hall physics

bandstructure in a solid, e.g. in GaAs:



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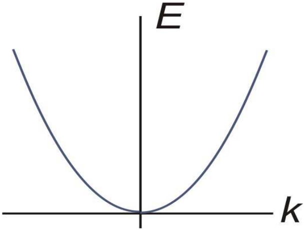

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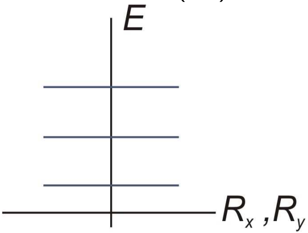
Landau levels (LL)



- $E = \hbar\omega(n + \frac{1}{2})$
- degeneracy $eB/h = 1/(2\pi\ell_0)^2$ (per unit area)
- filling factor ν : $\dfrac{\text{density of electrons}}{\text{LL degeneracy}}$

Filling factors with spin degree of freedom:

Non-interacting picture (only LLs)

two LL ladders offset by

$$E_Z = \mu_B g B \approx \frac{1}{60}\hbar\omega$$

filling factor:

$$\nu = \frac{N/A}{eB/h}$$

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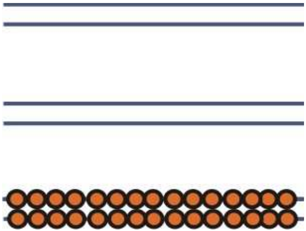

$\nu = 2$

Filling factors with spin degree of freedom:

Non-interacting picture (only LLs)

two LL ladders offset by

$$E_Z = \mu_B g B \approx \frac{1}{60}\hbar\omega$$

filling factor:

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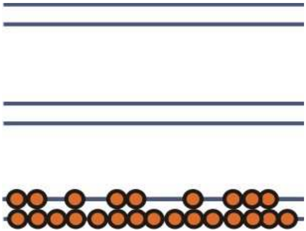

$\nu \approx 1.5$

Filling factors with spin degree of freedom:

Non-interacting picture (only LLs)

two LL ladders offset by

$$E_Z = \mu_B g B \approx \frac{1}{60}\hbar\omega$$

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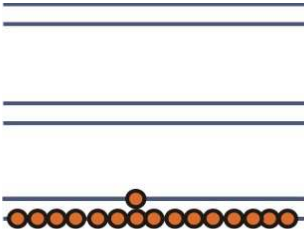

$$\nu = 1 + \varepsilon$$

Filling factors with spin degree of freedom:

Non-interacting picture (only LLs)

two LL ladders offset by

$$E_Z = \mu_B g B \approx \frac{1}{60}\hbar\omega$$

filling factor:

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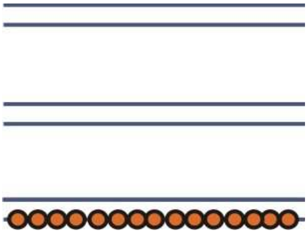

$\nu = 1$

Filling factors with spin degree of freedom:

Non-interacting picture (only LLs)

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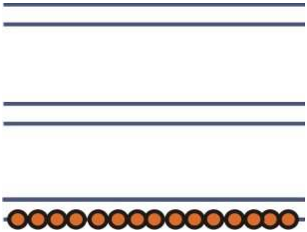
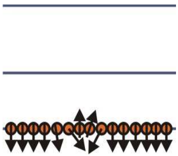
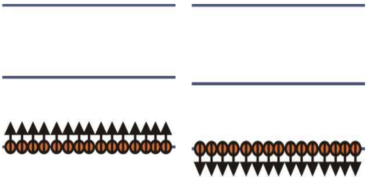


$\nu = 1$

Many possible GS configurations:



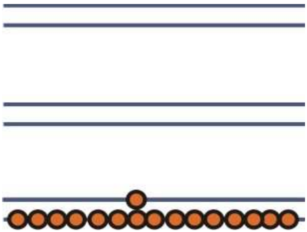
el-el interaction favours all spins aligned:



. . . or any other orientation

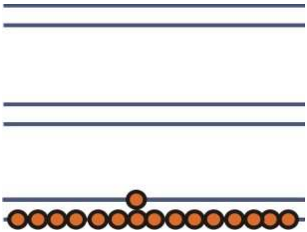
$\Rightarrow$ Heisenberg ferromagnet

Is there anything better than $\nu = 1$+QEr?



Is there anything better than $\nu = 1$+QEr?

binding an exciton to the QEr



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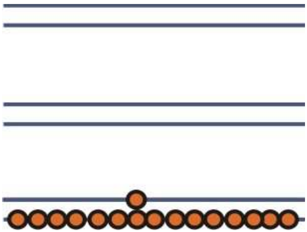

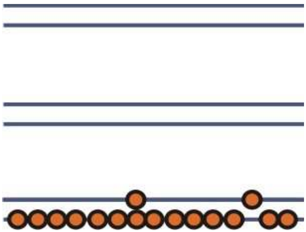

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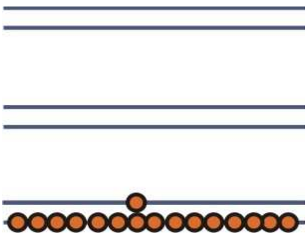

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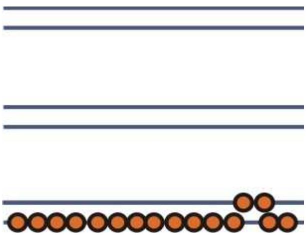

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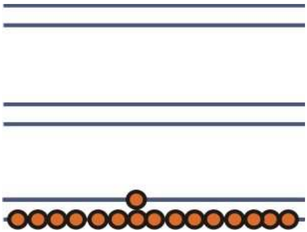

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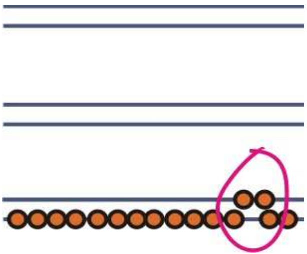

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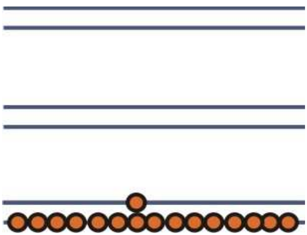
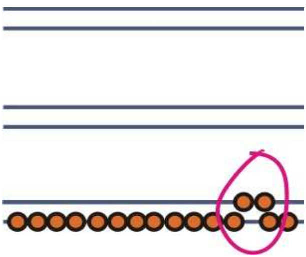
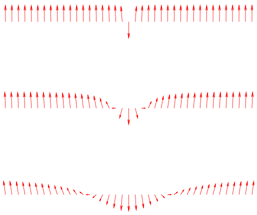


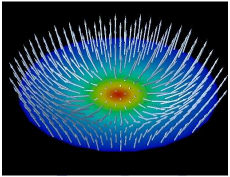




QEr

small skyrmion

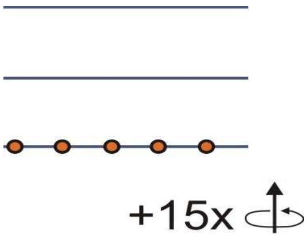
large skyrmion



Skyrmions in the fractional QHE

$\nu = \frac{N/A}{eB/h} = \frac{\#\text{ of els.}}{\#\text{ of fluxes}}$

$\nu = 1/3$ of electrons



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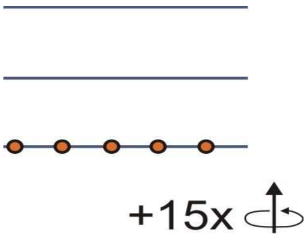
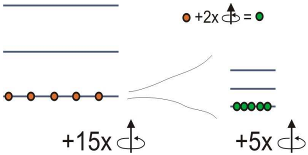


$\nu = 1$ of composite fermions



'trial wavefunctions'

Skyrmions in the fractional QHE

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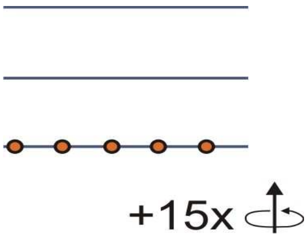

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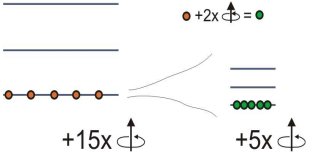
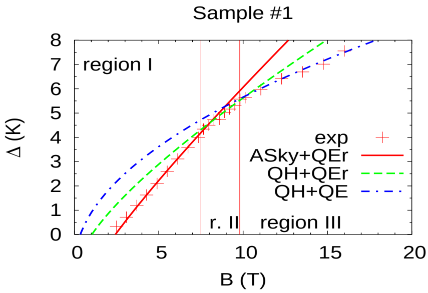


'trial wavefunctions'

Filling factor $\nu = 1/3$



activation gap vs magnetic field (energies in K, mag. field in T)

$$E_g = 50 E_C \sqrt{B} + 0.3\, B \times \Delta s_z + E_d$$

Dethlefsen et al., Phys. Rev. B '06

Summary

- microscopic model of how resistance of GaMnAs depends on the direction of magnetisation (AMR)

  Rushforth et al., Phys. Rev. Lett. '07, J. Mag. Magn. Mater. '08; de Ranieri et al., New J. Phys. '08

- identification of spin textures (in particular skyrmions) in experiments on fractional quantum Hall systems

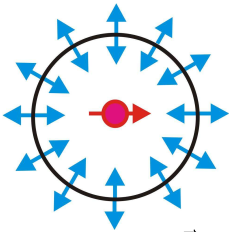
  Dethlefsen et al., Phys. Rev. B '06; KV et al., Phys. Rev. B '07; KV, Ann. Phys. (Leipzig) '07

. . .

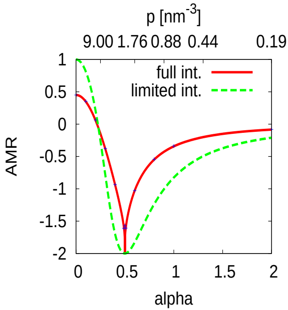
Non-crystalline AMR in a nutshell

Rushforth et al., JMMM '08

$$\sigma_{||} \propto \tau_x^+ + \tau_x^- = \frac{1}{(\alpha - \frac{1}{2})^2} + \frac{1}{(\alpha + \frac{1}{2})^2}, \qquad \sigma_\perp \propto 2\tau_y^\pm = \frac{2}{\alpha^2 + \frac{1}{12}}$$



$$V(k,k') + J_{pd}\vec{M}\cdot\vec{s}$$
$$\downarrow$$
$$\alpha + s_x$$



coherent sum of electric (Mn = ionized acceptor) and magnetic (Mn magnetic moment) scattering op.
α = effective strength/electric part

sign change!

$$\text{AMR}/2 = -\frac{\sigma_{||} - \sigma_\perp}{\sigma_{||} + \sigma_\perp}$$