

# Effective model of antiferromagnetic MnTe and anisotropic magnetoresistance

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KCL

Technical Univ.  
Eindhoven



TUe

University  
Regensburg



UR

solid state physics

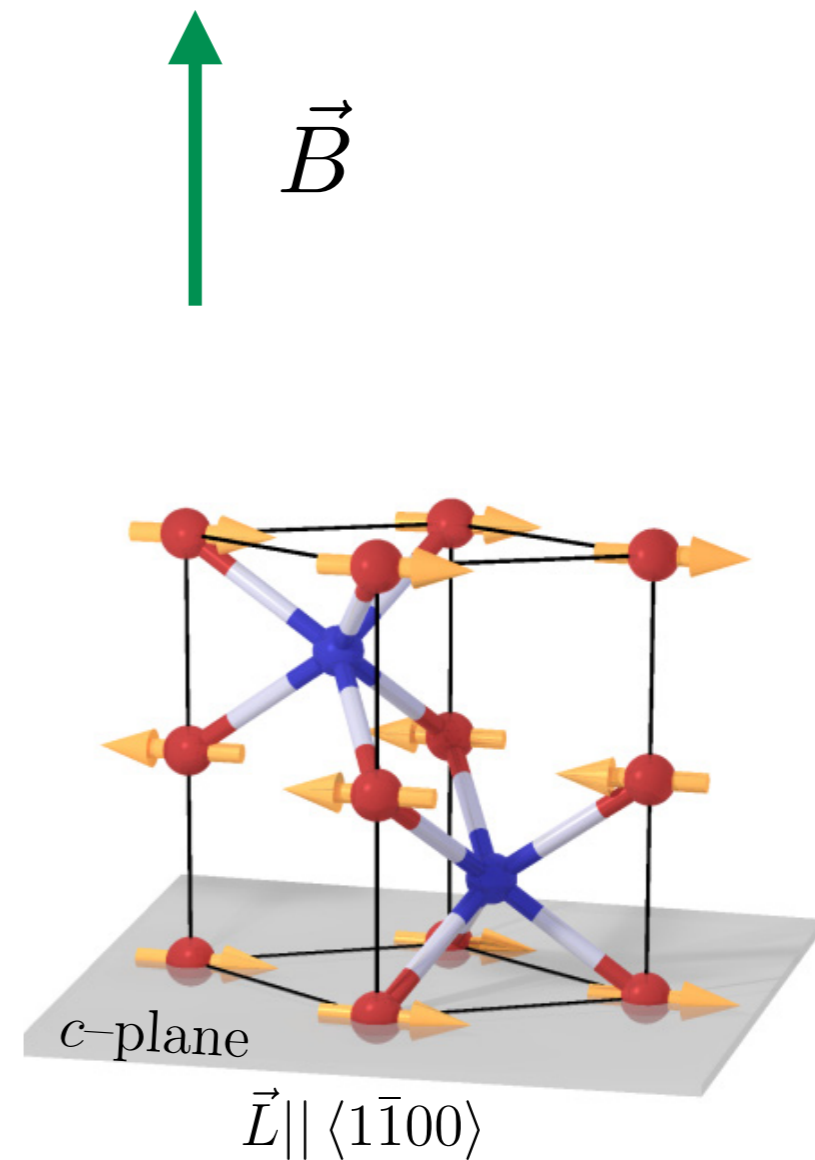
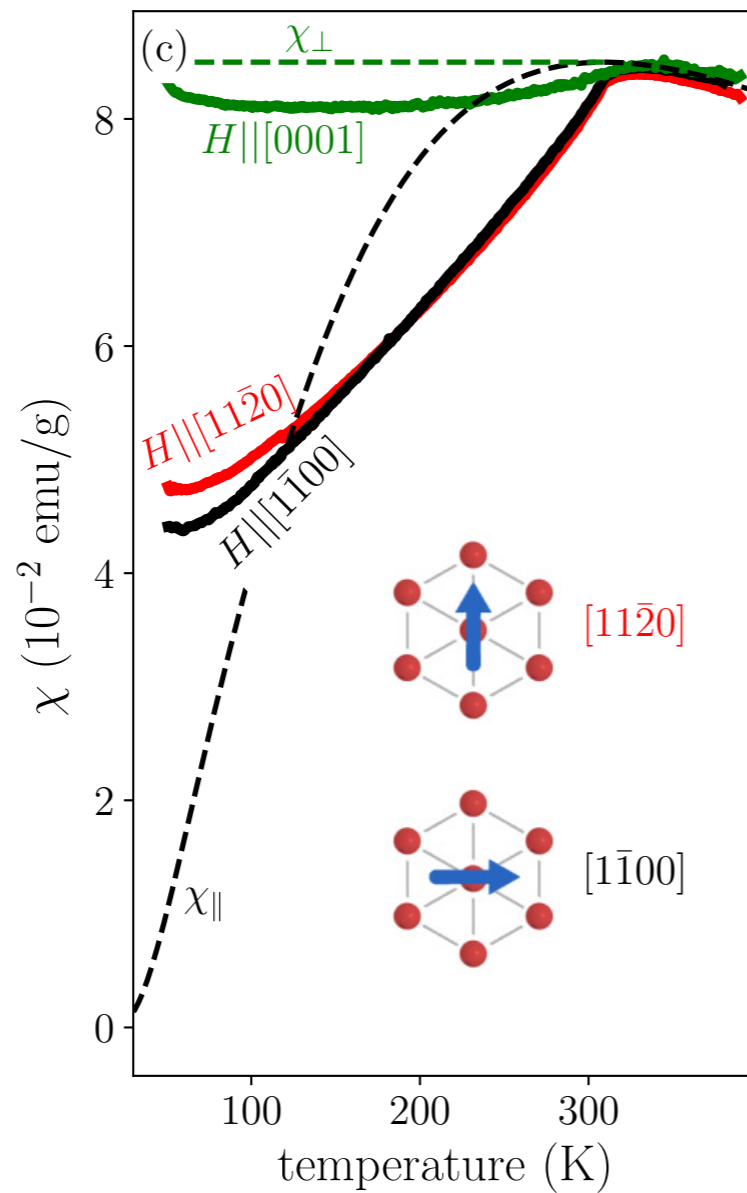
materials science

anisotropic  
magnetoresistance  
(AMR)

iron, permalloy, (Ga,Mn)As  
antiferromagnets

# MnTe: an antiferromagnet

## Magnetometry

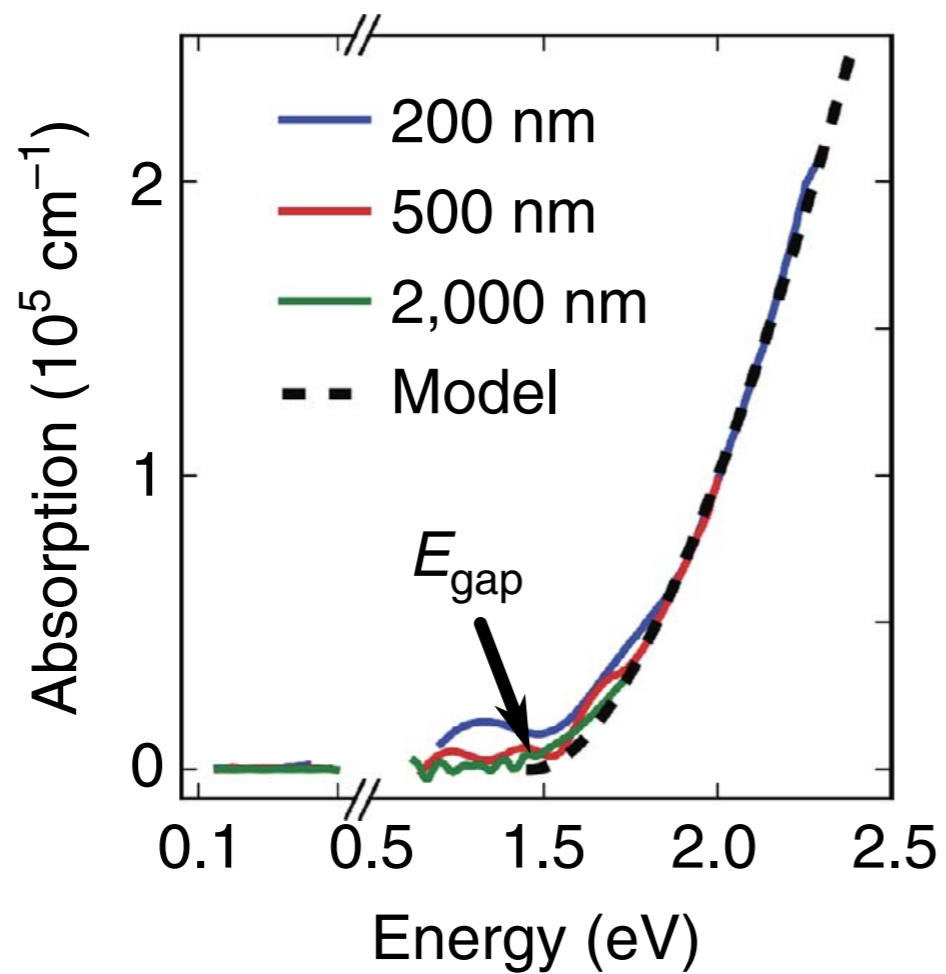


Kriegner et al. '17

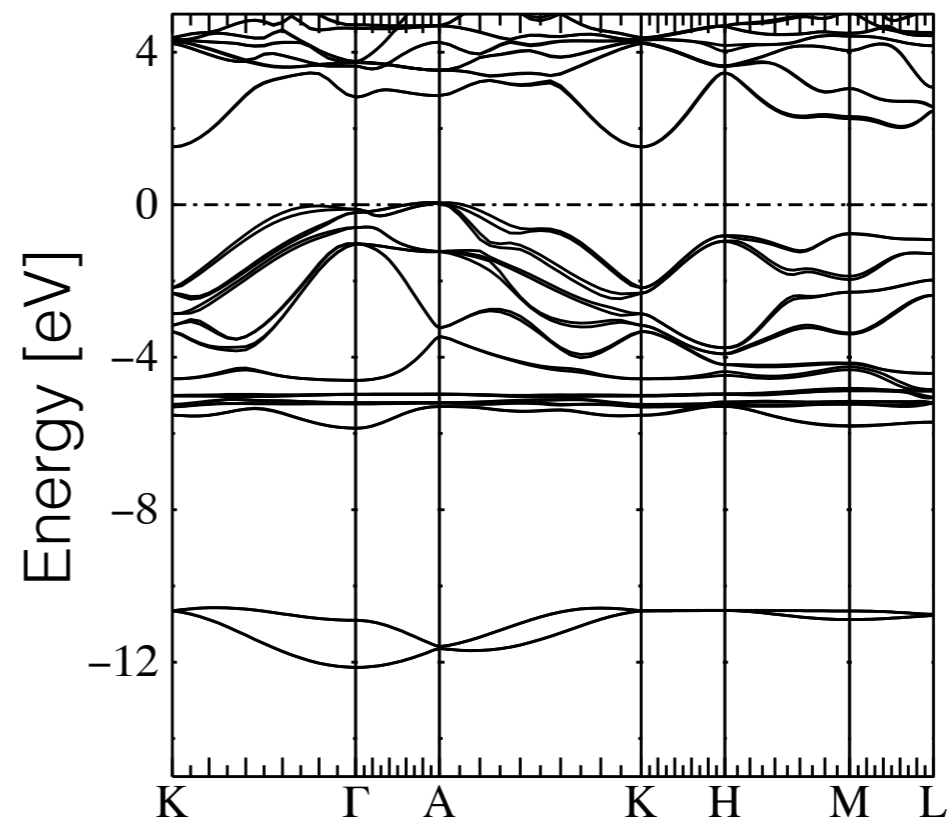
doi: 10.1103/PhysRevB.96.214418

# MnTe: a semiconductor

absorption on thin layers



QSGW

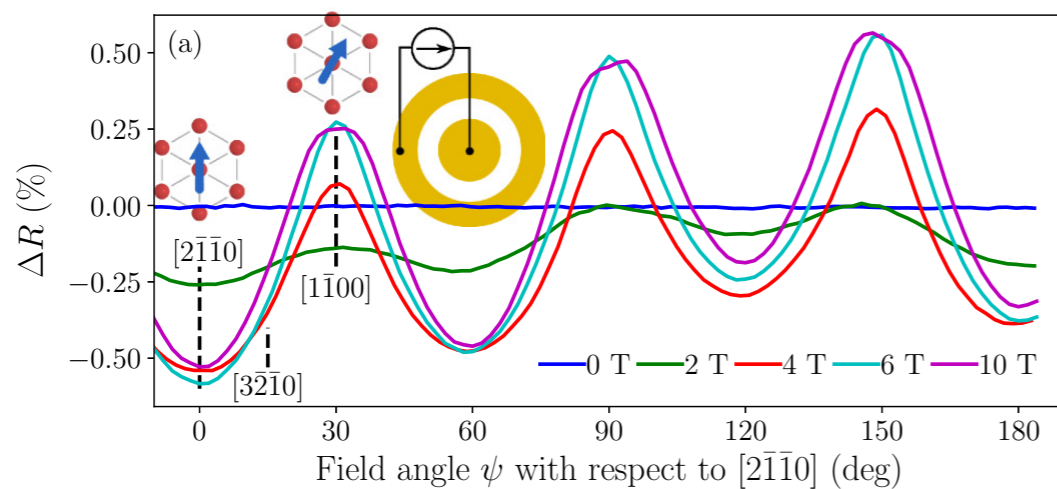


Kriegner et al. '16  
doi: 10.1038/ncomms11623

# The goal:

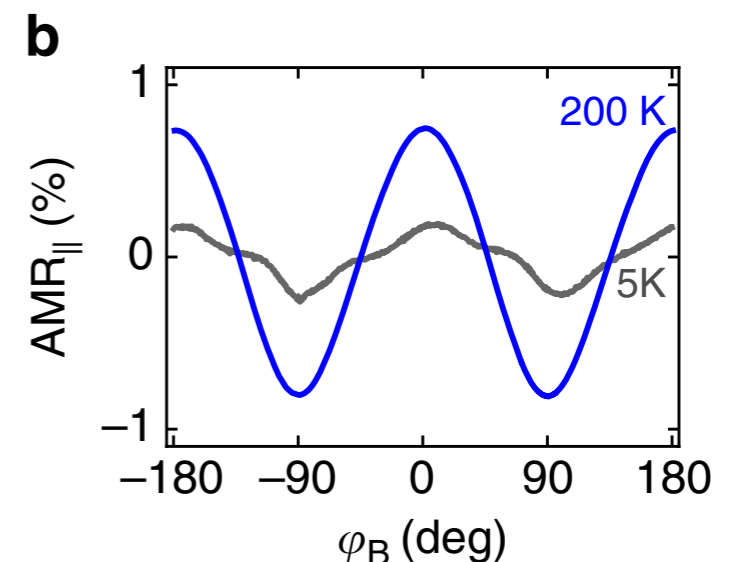
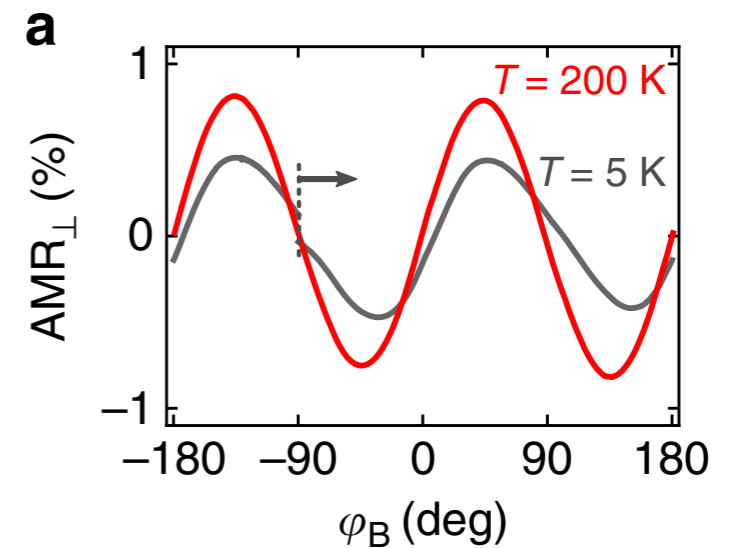
- construct an effective model
- analyse magnetic anisotropies
- AMR modelling

- non-crystalline components (Hall)
- crystalline components (Corbino)



Kriegner et al. '17

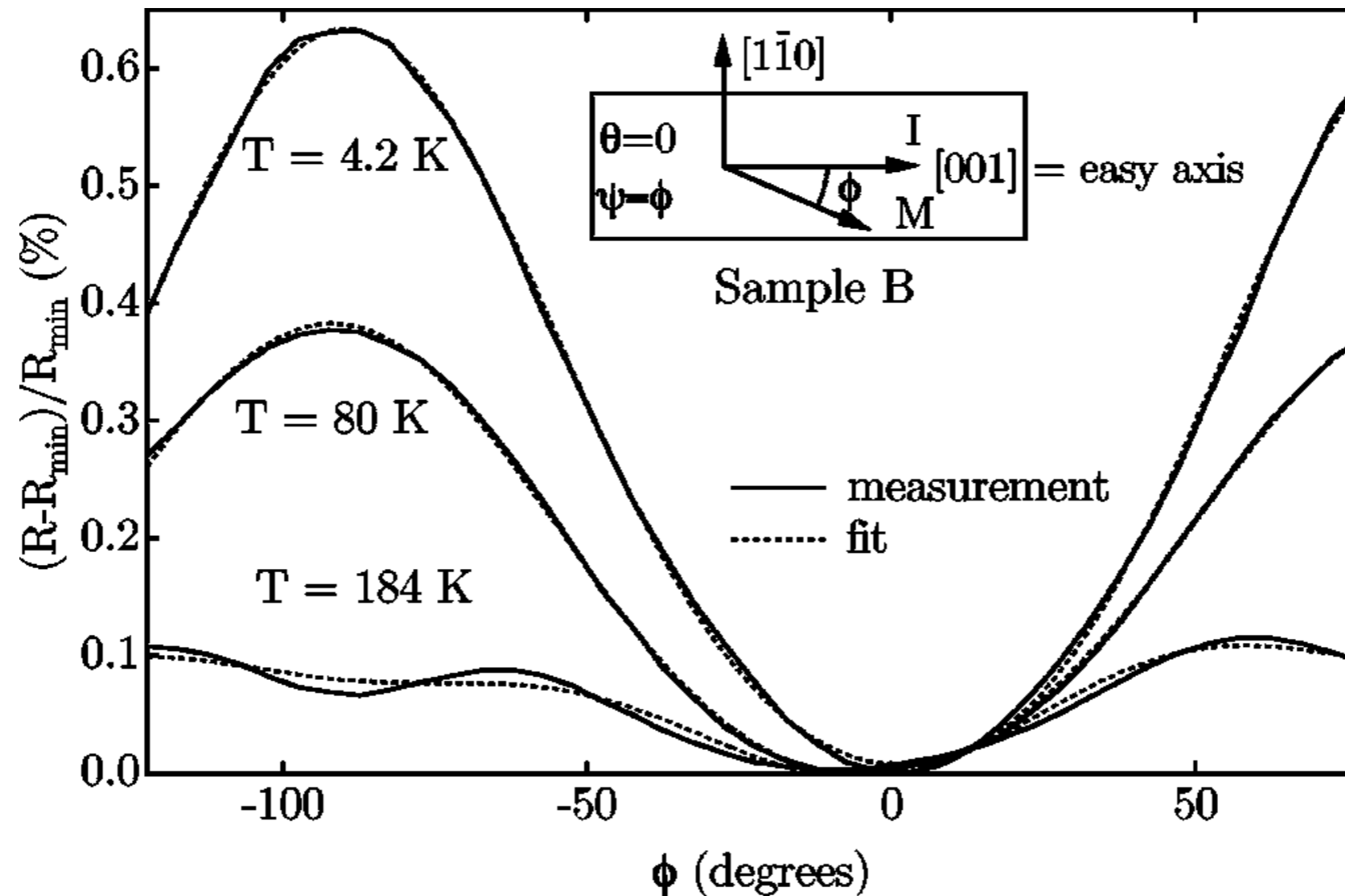
doi: 10.1103/PhysRevB.96.214418



Kriegner et al. '16

doi: 10.1038/ncomms11623

# AMR: an “old” phenomenon



- discovered in mid 19th century
- more recently: epitaxial films (here, Fe)

van Gorkom et al. '01

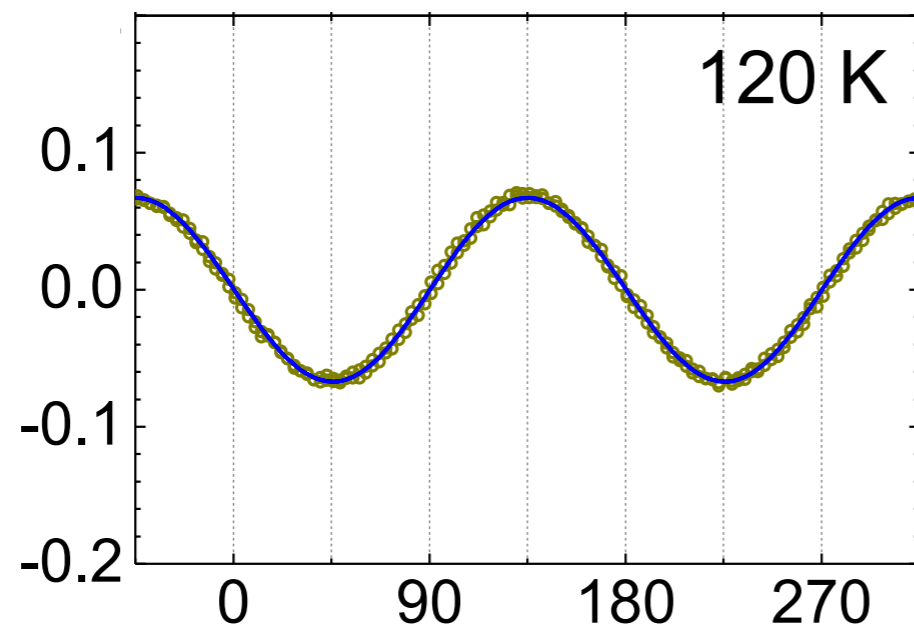
doi: 10.1103/PhysRevB.63.134432

AMR: an “old” phenomenon - still explored  
 (Ga,Mn)As - ferromagnetic semiconductor, here ~10% Mn

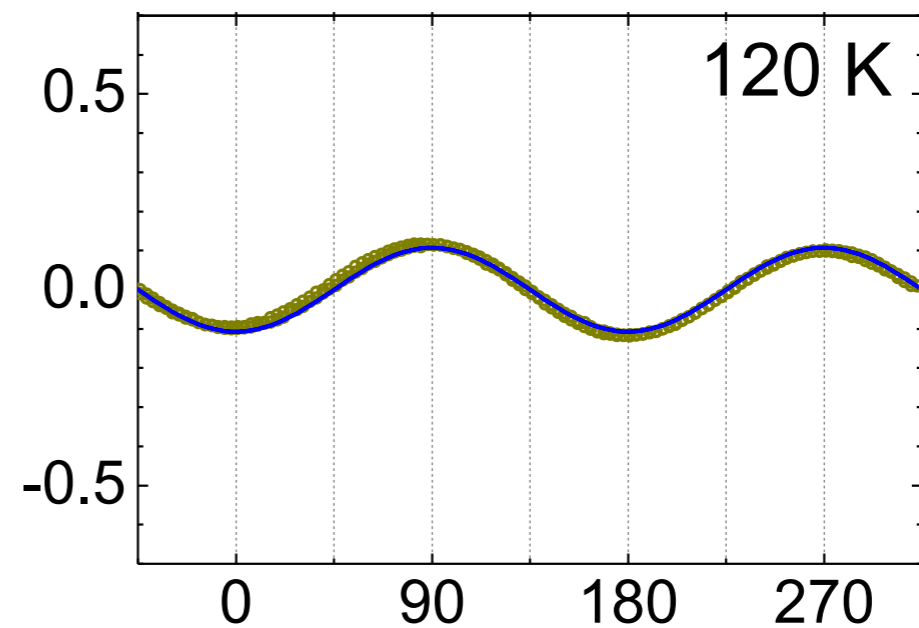
longitudinal:  $R_{\parallel}$

transversal:  $R_{\perp}$

$\Delta R_{xx}/R_{av}$



$R_{xy}/R_{av}$



In-plane magnetic field angle,  $\phi_H$  (deg)

$$\phi = \phi_H + \theta, \quad \theta = 45^\circ$$

Miyakozawa et al. '16

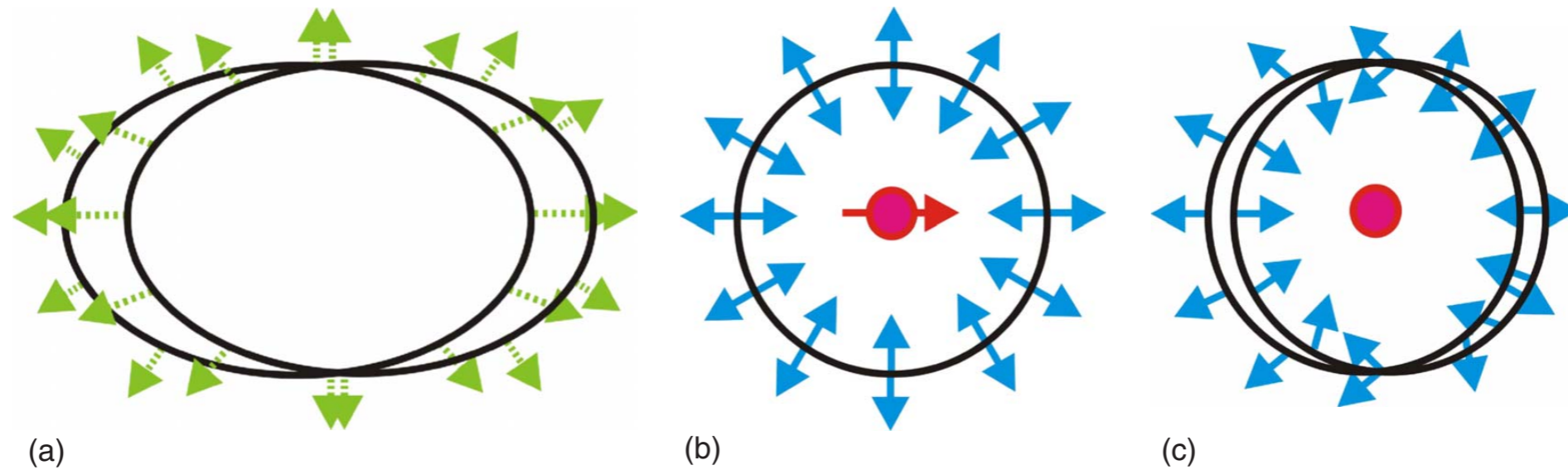
10.1063/1.4944328

$$\Delta\rho_{xx}/\rho_0 = C_I \cos 2\phi$$

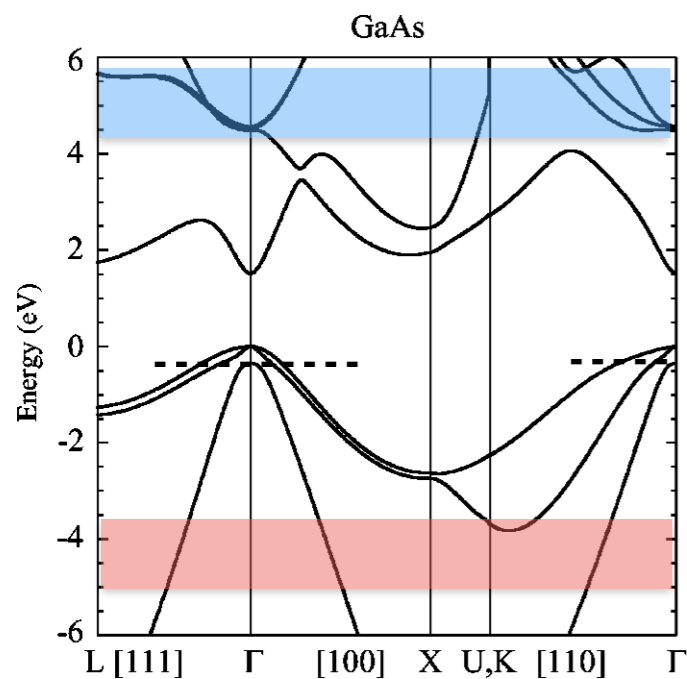
$$\rho_{xy}/\rho_0 = C_I \sin 2\phi$$



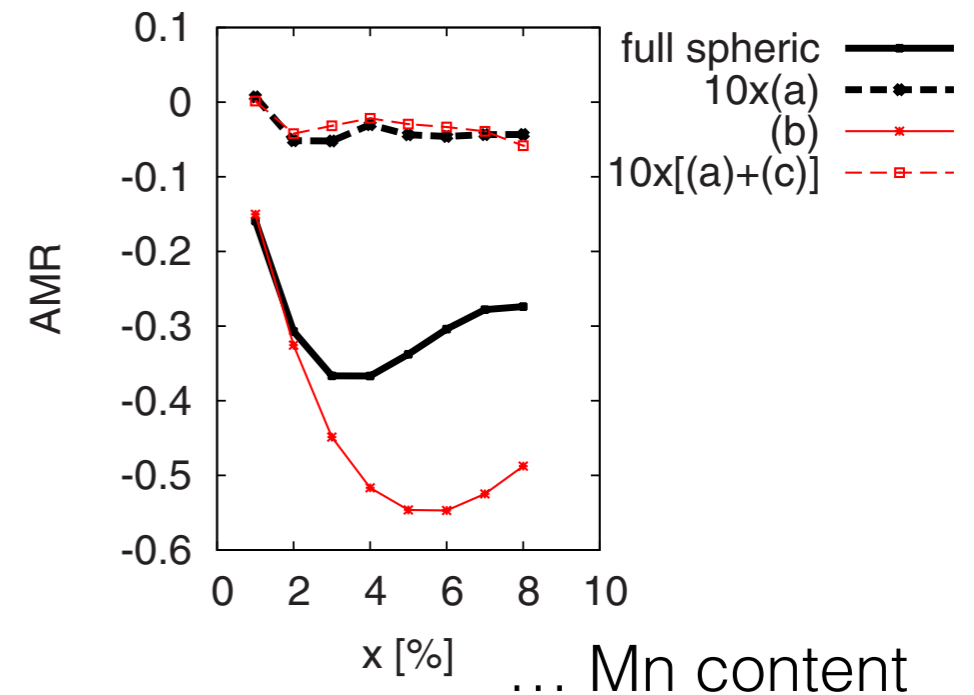
# AMR: the idea of analysis



example: ferromagnetic (Ga,Mn)As



- Mn dopants: both magnetic and non-mag. char.
- VB: p-states of As
- Mn 3d states split the VB top



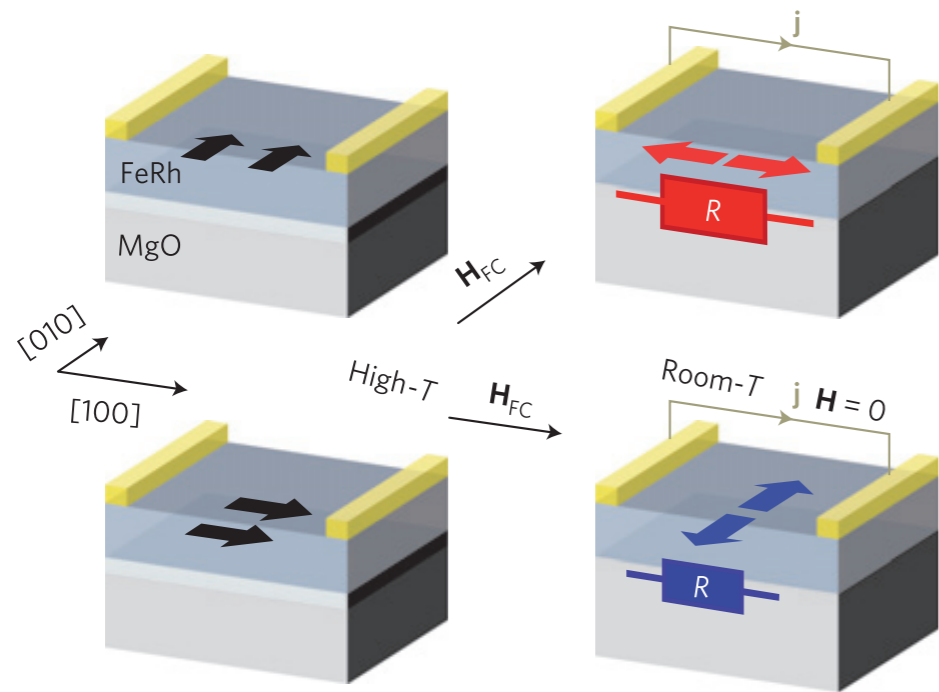
$$H = H_{KL} + (J_{pd}/\mu_B)\vec{M} \cdot \vec{s}$$

the humble myself et al. '09

doi: 10.1103/PhysRevB.80.165204

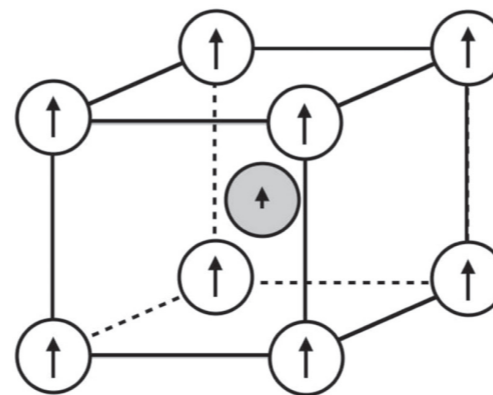
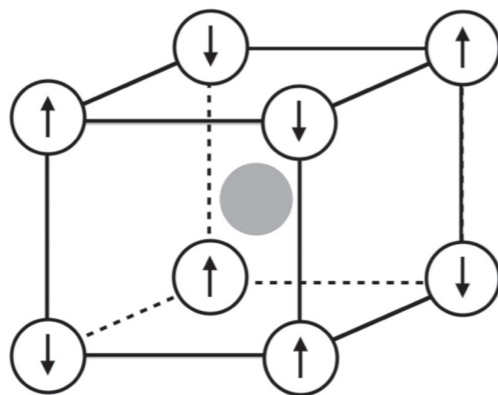
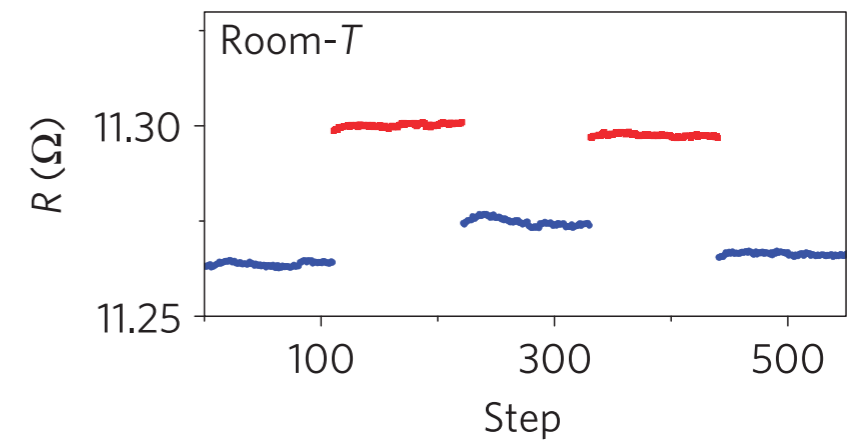
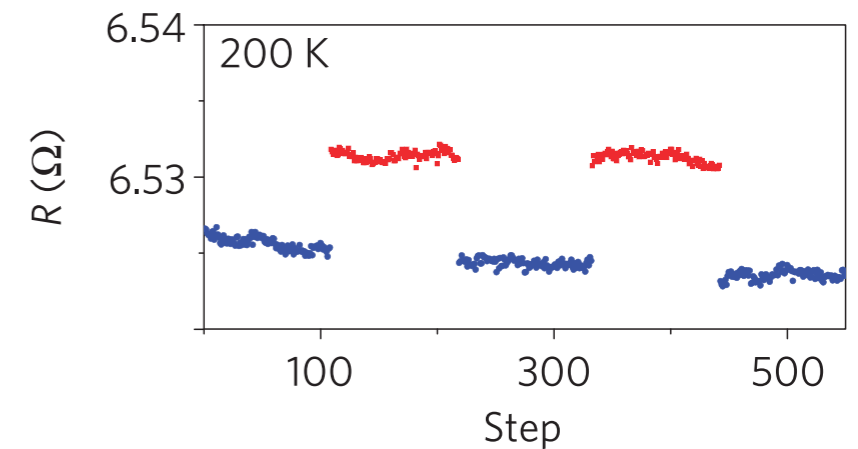
AMR: the new twist

# AMR: the new twist antiferromagnets!

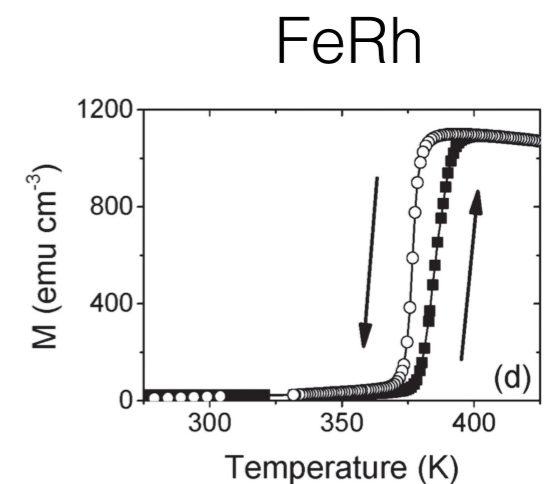


- AFM at low T,  
FM at higher T
- cool-down in  
magnetic field
- 'memory',  
read-out via AMR

Marti et al. '14  
10.1038/nmat3861



**Figure 5.** Structure of bulk FeRh. In the AF phase (left), the Rh atom (grey) has no magnetic moment while the Fe magnetic moments are ordered as indicated. Except for a different lattice constant (see text), the crystal structure remains unchanged in the FM phase (right); magnetic moment of the Rh atoms is then non-zero and parallel with that of Fe atoms.



Saidl et al. '15

10.1088/1367-2630/18/8/083017

# MnTe in hexagonal NiAs structure

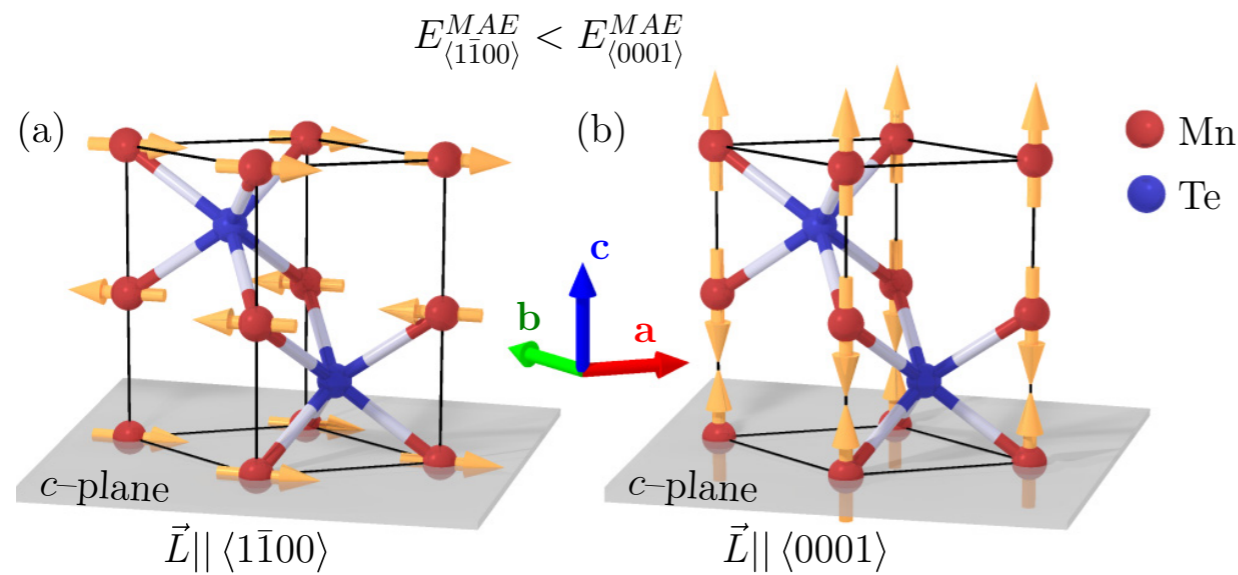
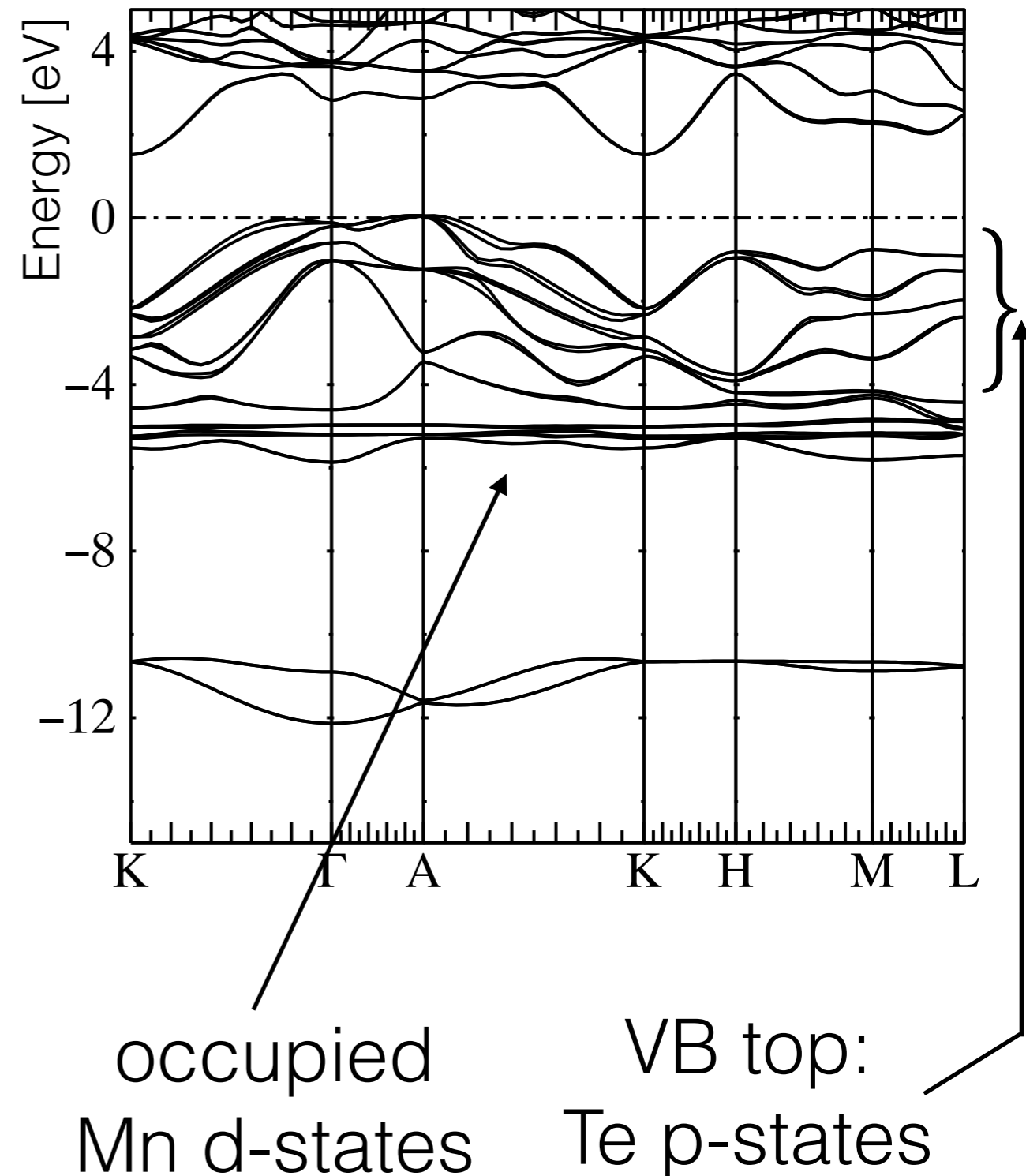
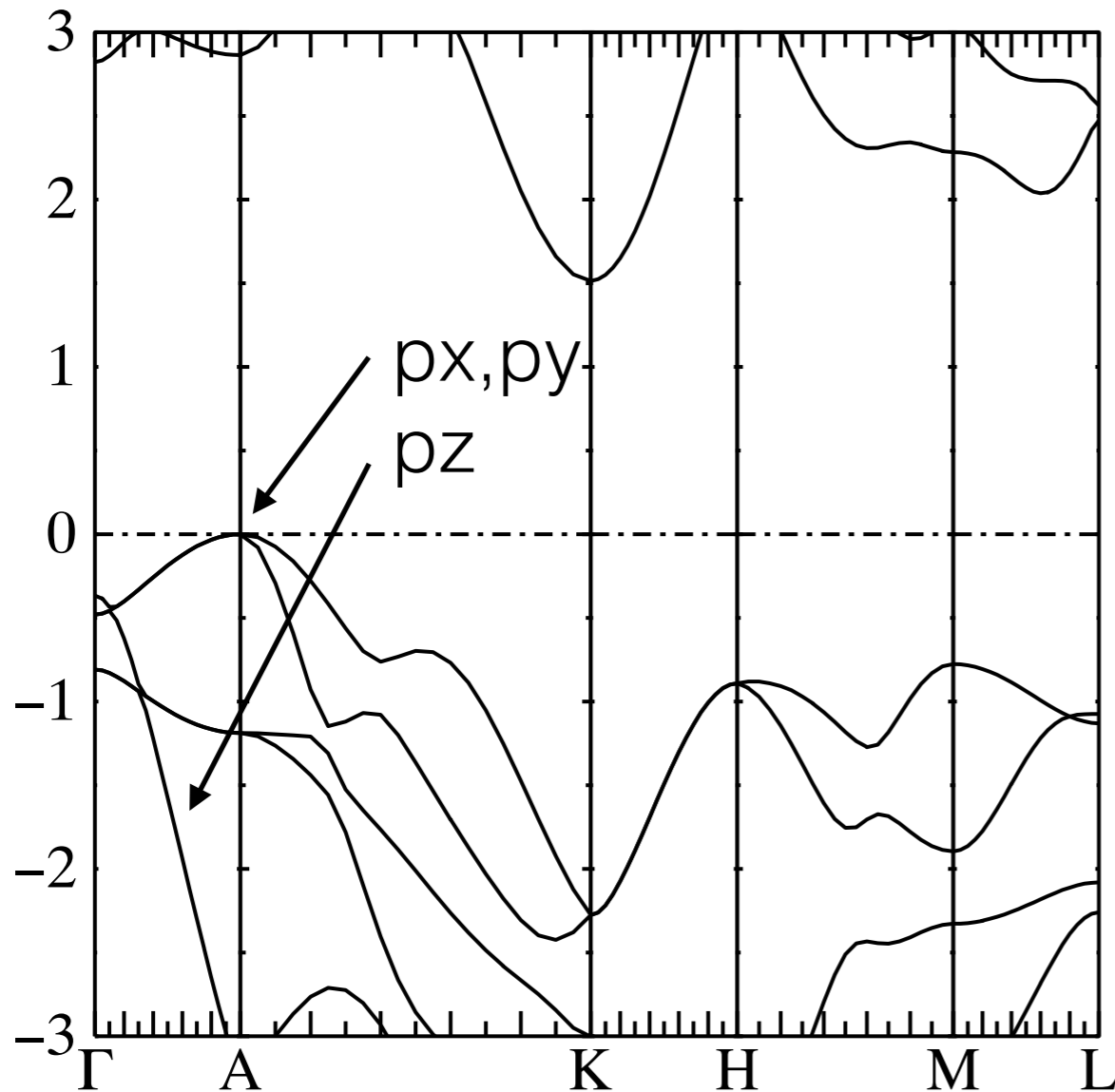


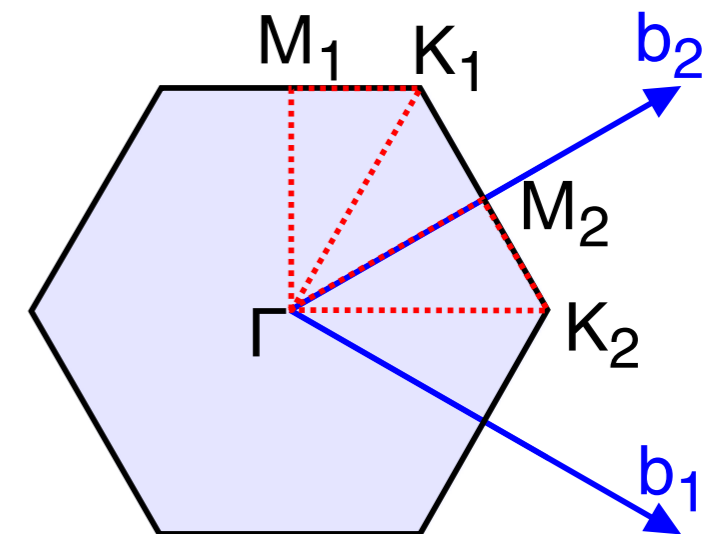
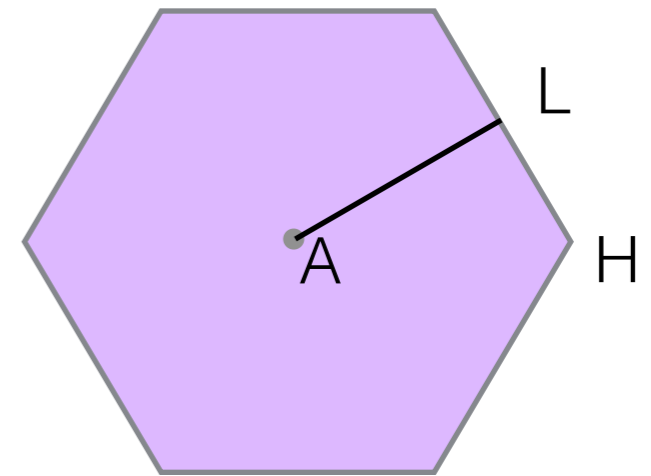
FIG. 1. Sketch of the atomic and possible magnetic structures of antiferromagnetic hexagonal MnTe. (a) In-plane/*c* plane (ground state) and (b) out-of-plane/*c*-axis (hard axis) orientation of the magnetic moments of Mn with the Néel vector  $\vec{L}$  along  $\langle 1\bar{1}00 \rangle$  and  $\langle 0001 \rangle$  are shown. The hexagonal basal plane, i.e., the *c* plane is indicated by a gray plane, while red, green, and blue arrows show the directions of the unit cell axes.



# detail of the VB (no SO)

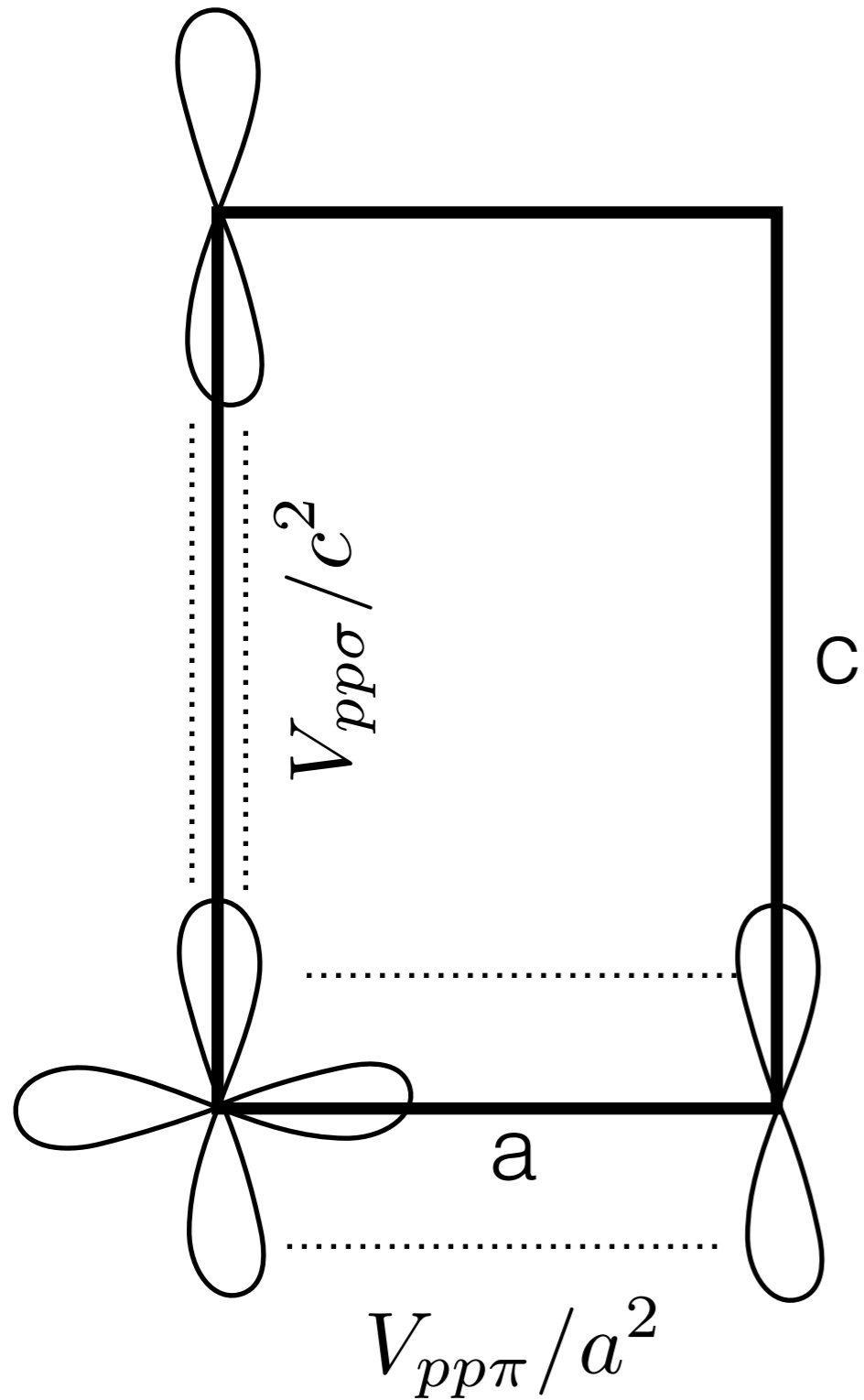


- warping in  $k_x, k_y$  plane neglected
- $d=0$  for simplicity (warping away from  $k_z=0$ )



$$H = \frac{\hbar^2}{2m_0} \begin{bmatrix} ak_x^2 + bk_y^2 + ck_z^2 + dk_yk_z & (a-b)k_xk_y + dk_xk_z \\ (a-b)k_xk_y + dk_xk_z & bk_x^2 + ak_y^2 + ck_z^2 - dk_yk_z \end{bmatrix}$$

competing maxima at the VB top: the idea

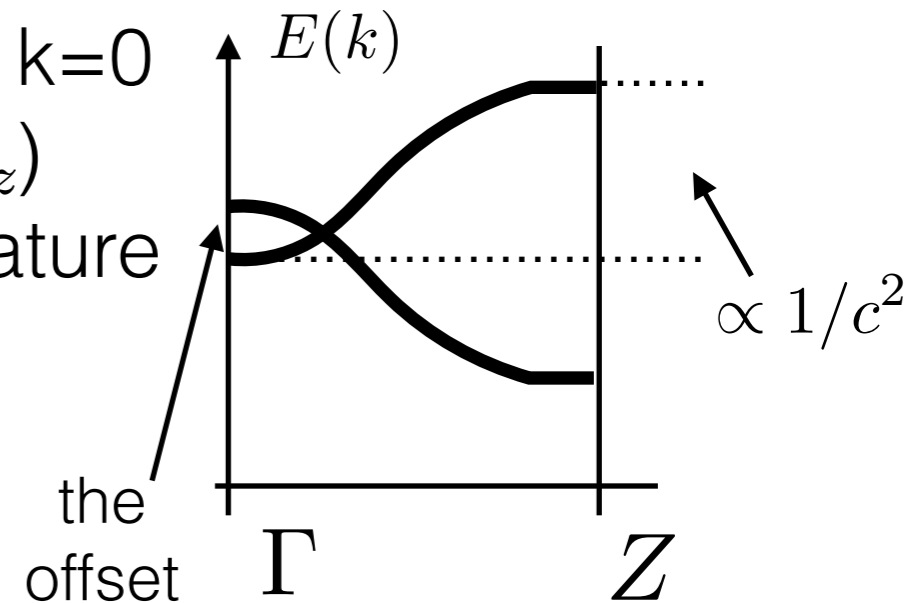


$$H = \begin{pmatrix} E_X & 0 \\ 0 & E_Z \end{pmatrix}$$

$$E_X = \frac{2V_{pp\pi}}{a^2} \cos ak_x + \frac{2V_{pp\sigma}}{c^2} \cos ck_z$$

$$E_Z = \frac{2V_{pp\sigma}}{a^2} \cos ak_x + \frac{2V_{pp\pi}}{c^2} \cos ck_z + \Delta_z$$

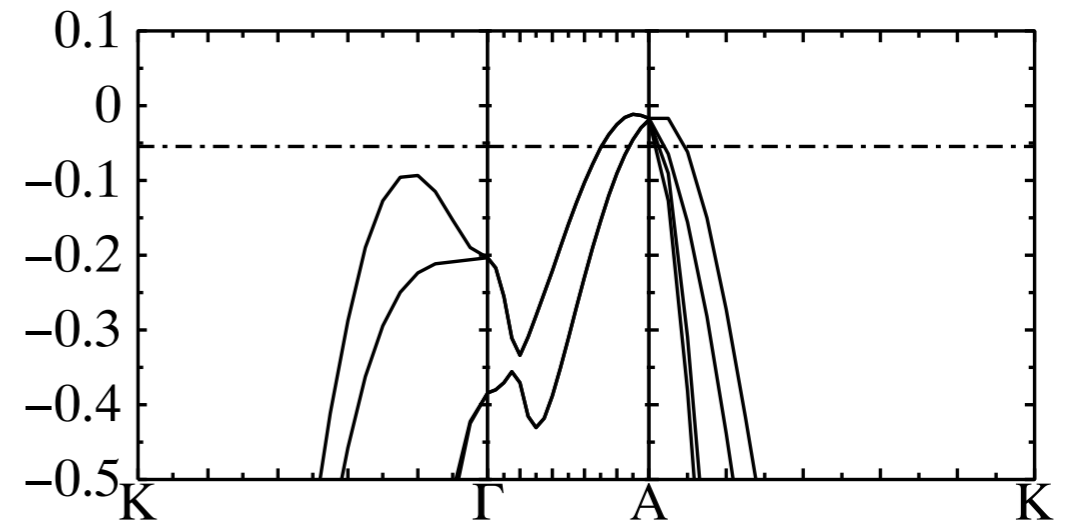
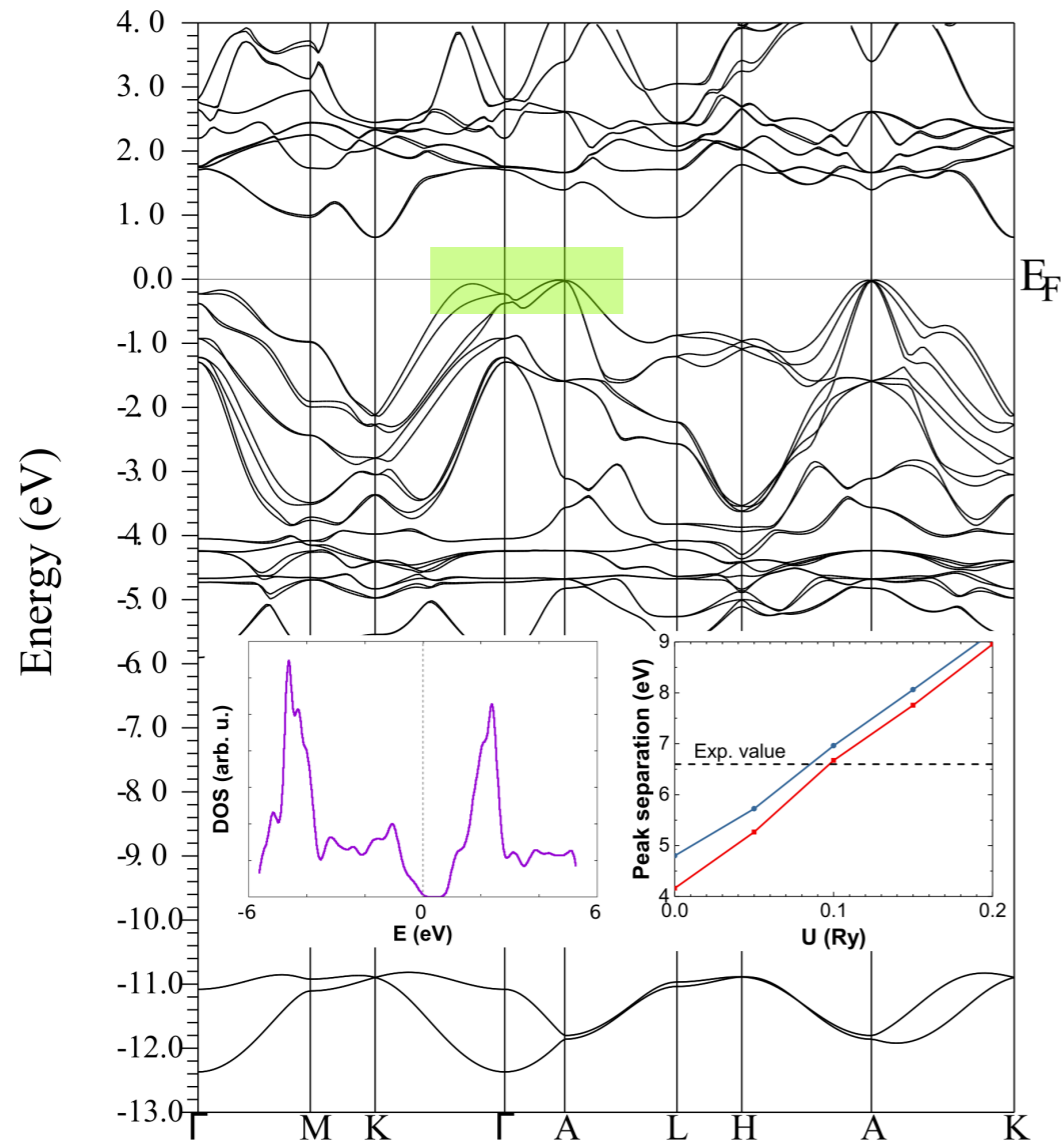
- degenerate at  $k=0$  for  $a=c$  (negl.  $\Delta_z$ )
- opposite curvature for  $V_{pp\sigma} V_{pp\pi} < 0$



# competing maxima at the VB top: *ab initio*

DFT+U

GW



type	$a/c$	position of max.
Ref. [1]	0.409/0.643	
bulk	0.414/0.671	around A
SrF <sub>2</sub> LT	0.4135/0.6655	around A
SrF <sub>2</sub> RT	0.4148/0.672	around A
InP RT	0.417/0.669	VBM1 $\approx$ VBM2
InP LT	0.417/0.660	$\Gamma$ -K line

how to deal with exchange

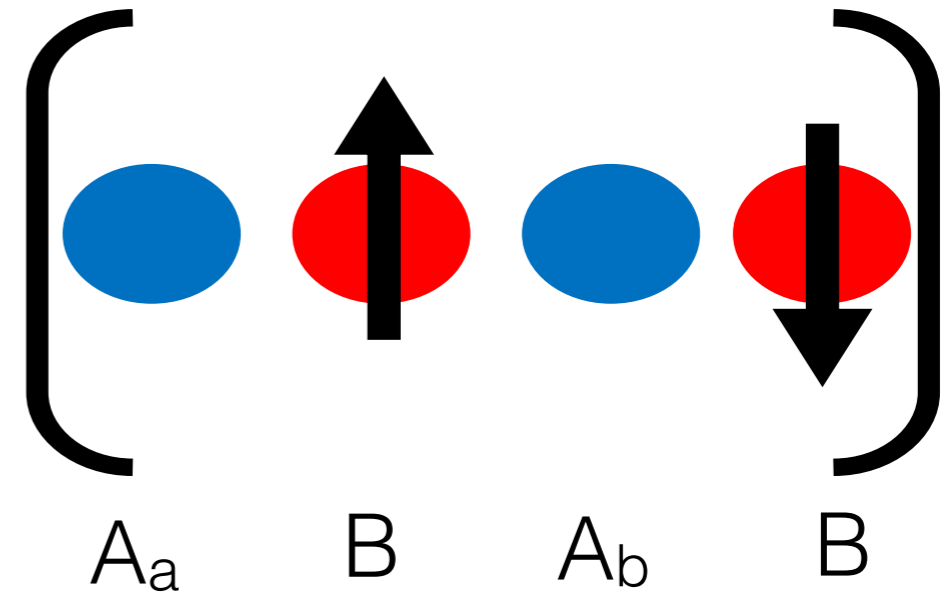
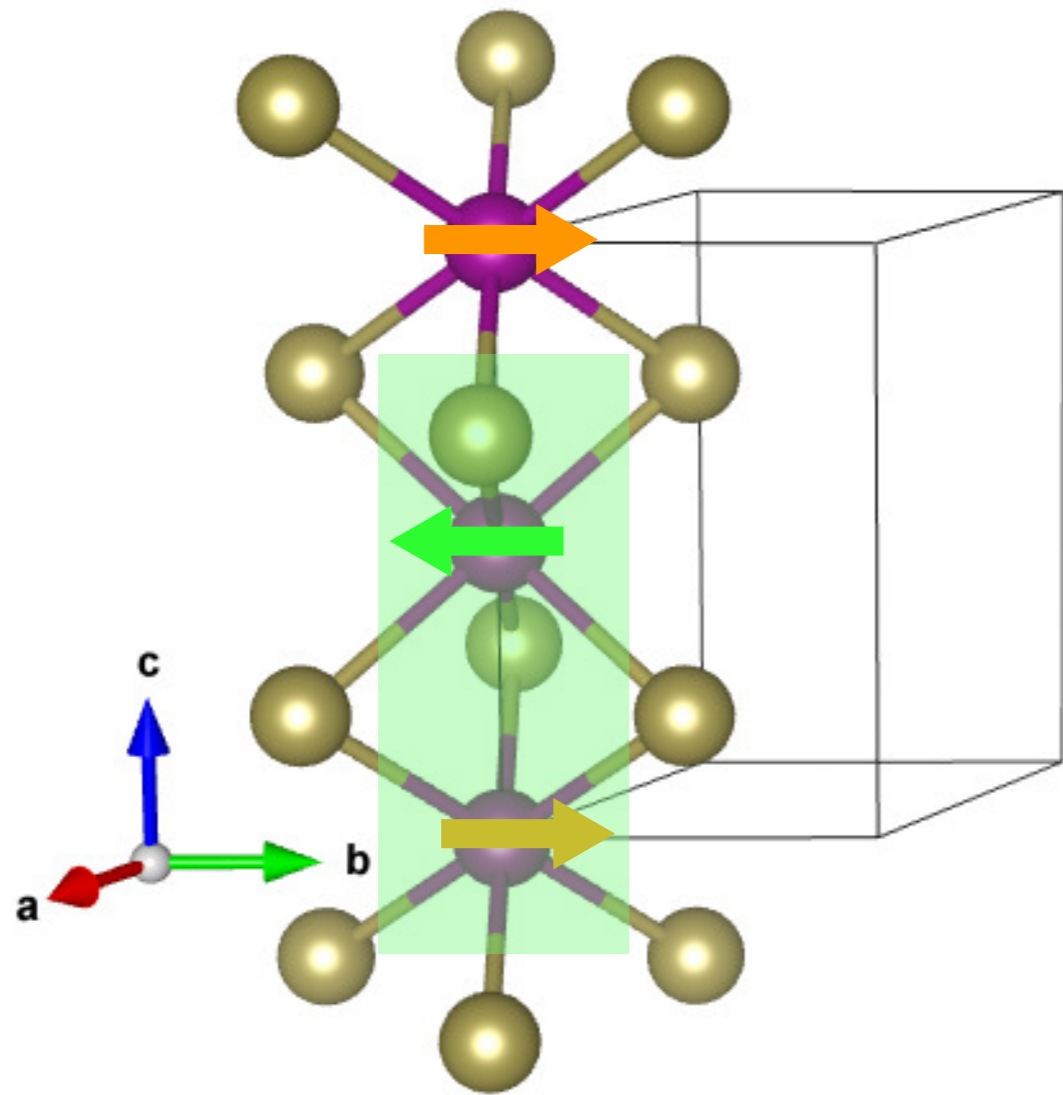
ferromagnetic semiconductor

$$H = H_{KL} + (J_{pd}/\mu_B)\vec{M} \cdot \vec{s}$$

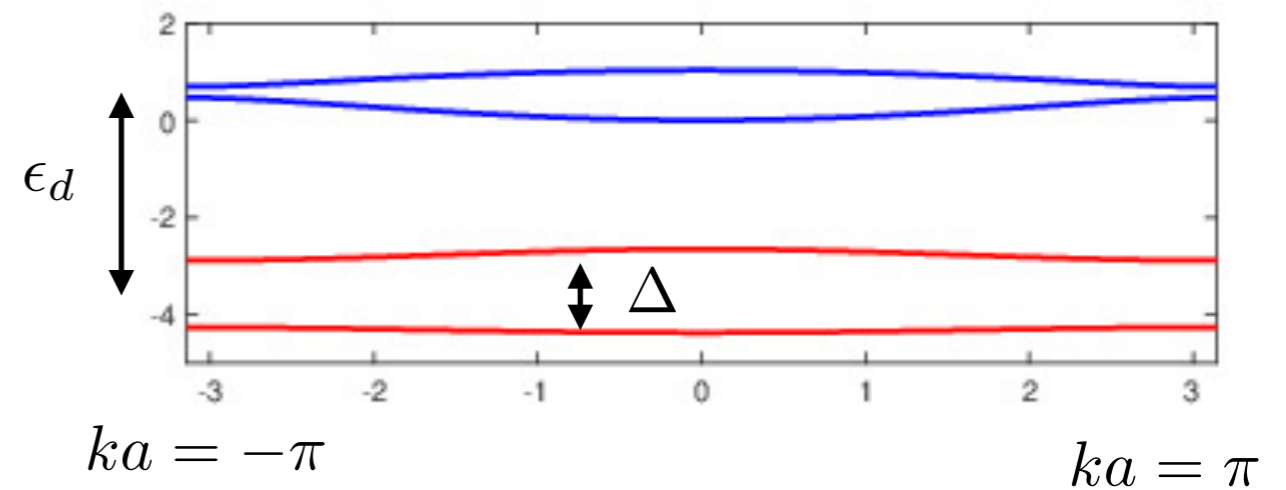
but in an antiferromagnet???



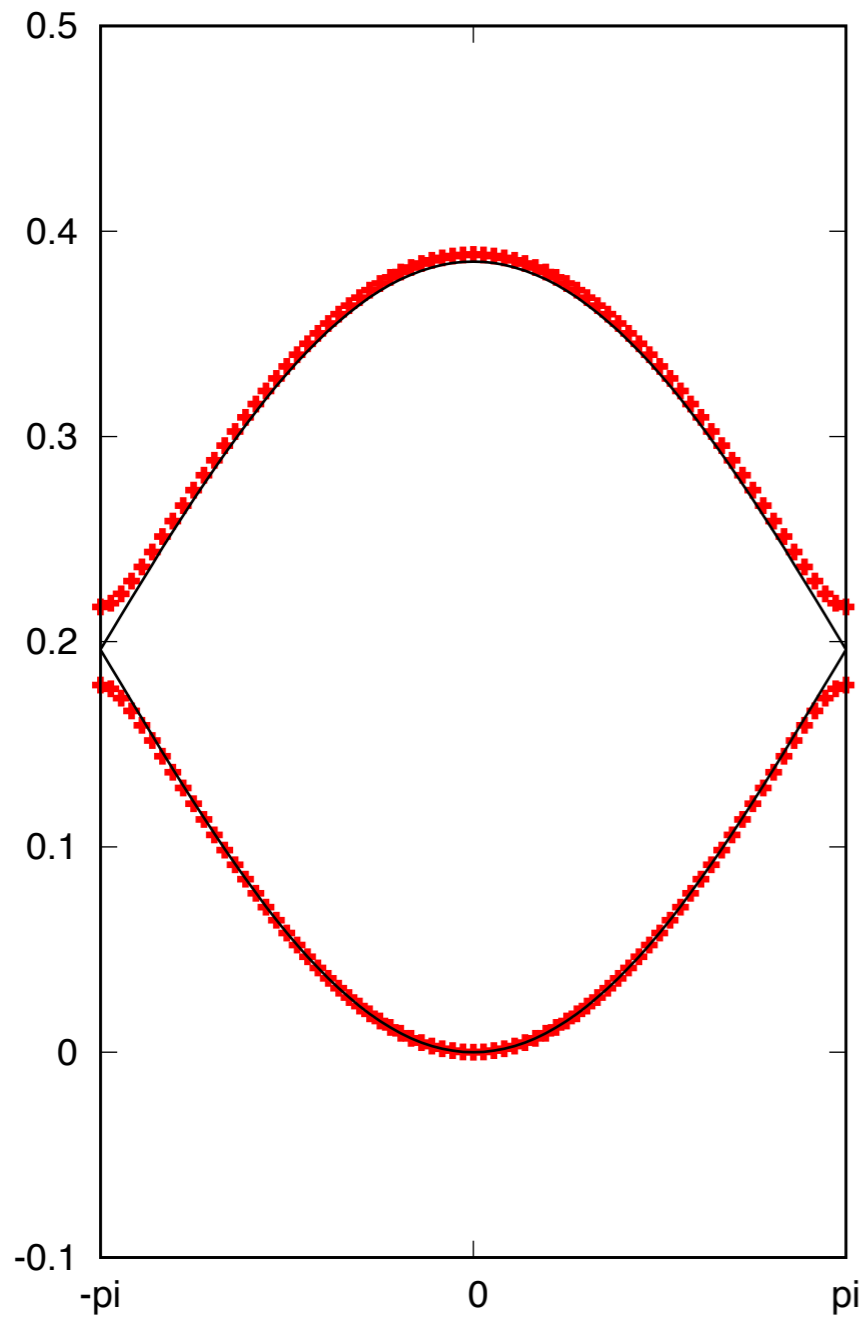
# how to deal with exchange - a toy model



$$H = \begin{pmatrix} 0 & t & 0 & te^{-ika} \\ t & \epsilon_d + \Delta & t & 0 \\ 0 & t & 0 & t \\ te^{ika} & 0 & t & \epsilon_d - \Delta \end{pmatrix}$$

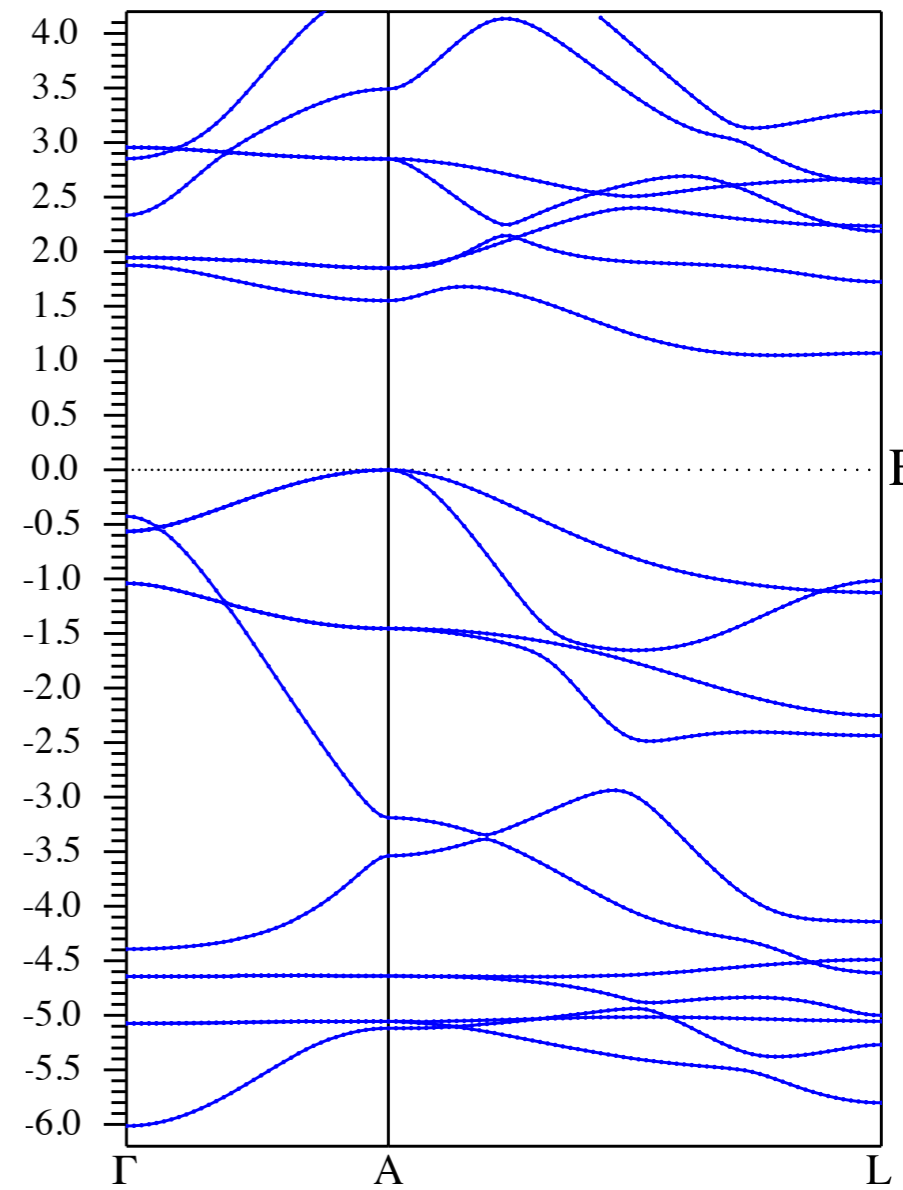


# how to deal with exchange - a toy model wavefunctions

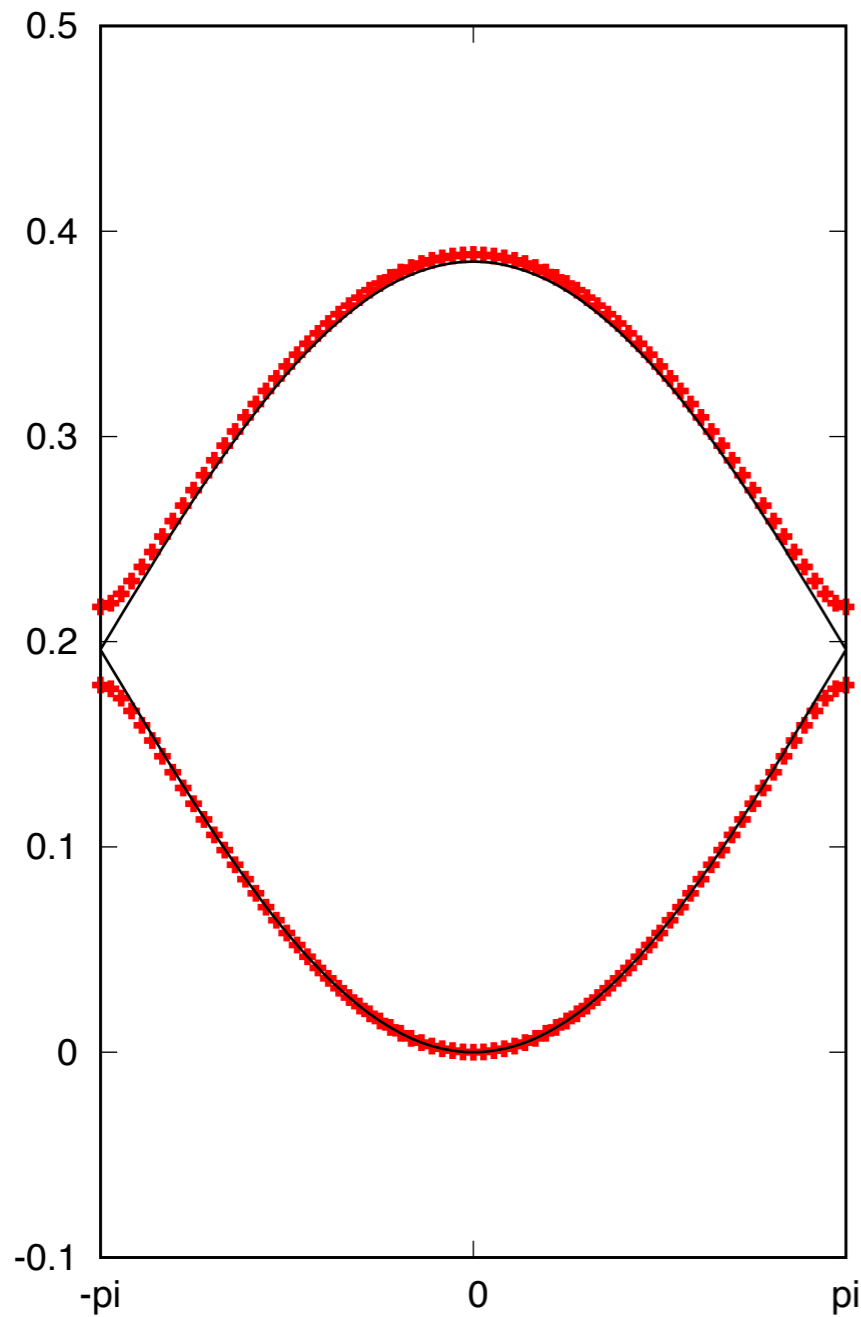


$$\begin{aligned}
 & (|a\rangle - |b\rangle) \otimes |\downarrow\rangle \\
 & (|a\rangle + |b\rangle) \otimes |\uparrow\rangle
 \end{aligned}$$

$A_a$ 
 $B$ 
 $A_b$ 
 $B$

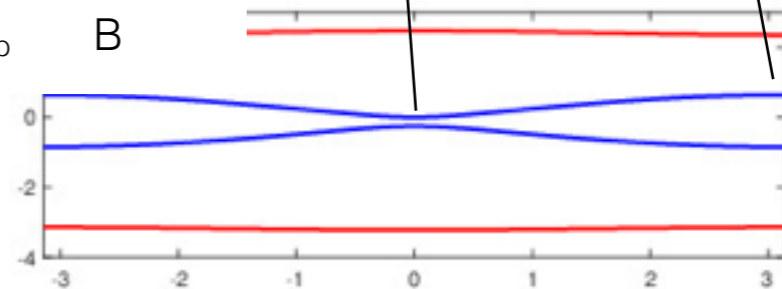
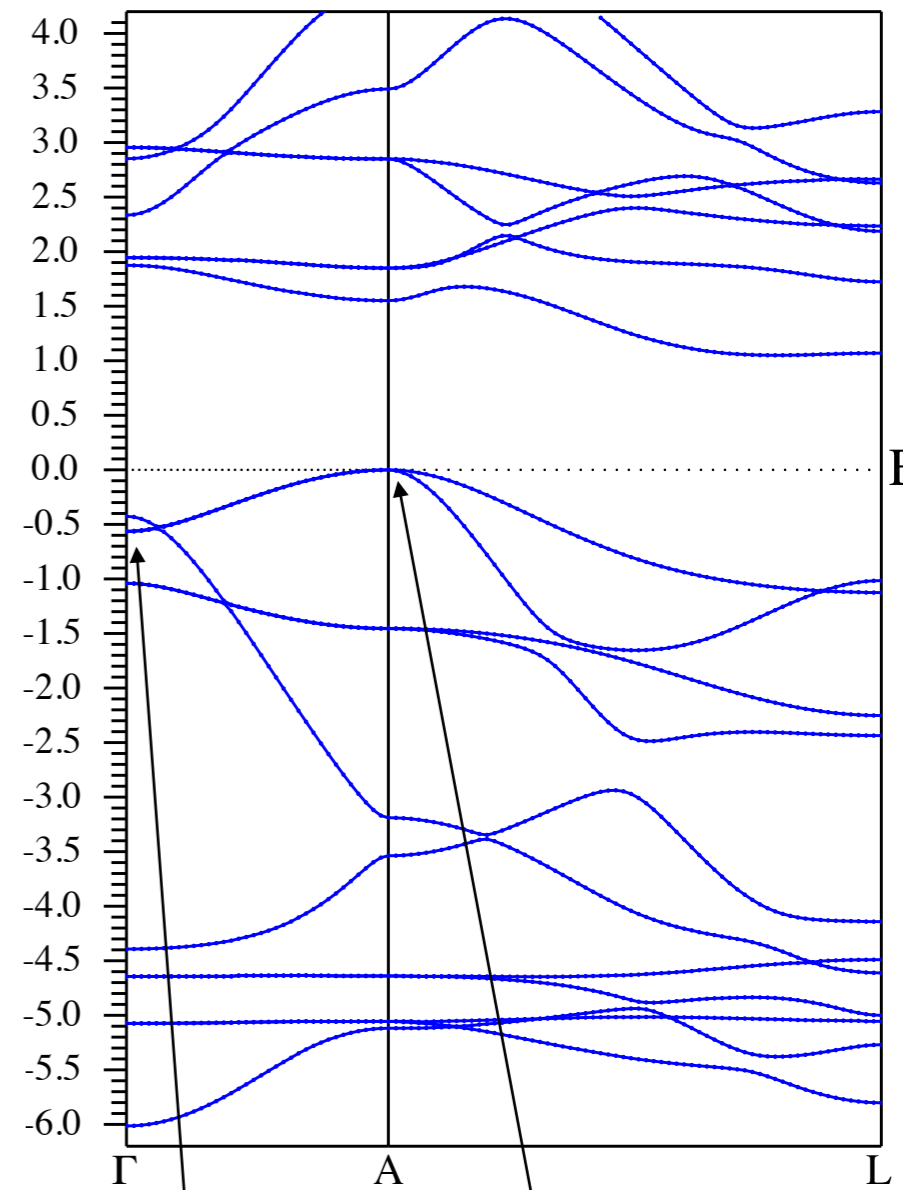
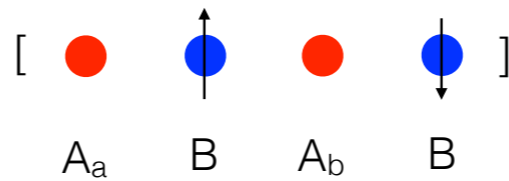


# how to deal with exchange - a toy model wavefunctions



$$(|a\rangle - |b\rangle) \otimes |\downarrow\rangle$$

$$(|a\rangle + |b\rangle) \otimes |\uparrow\rangle$$



$$\Delta > \epsilon_d$$

$$H = H_b + H_{so} + H_{ex}$$

changed to

$$H = H_{be} + H_{so}$$

& matrix els. of  $H_{so}$  recalculated

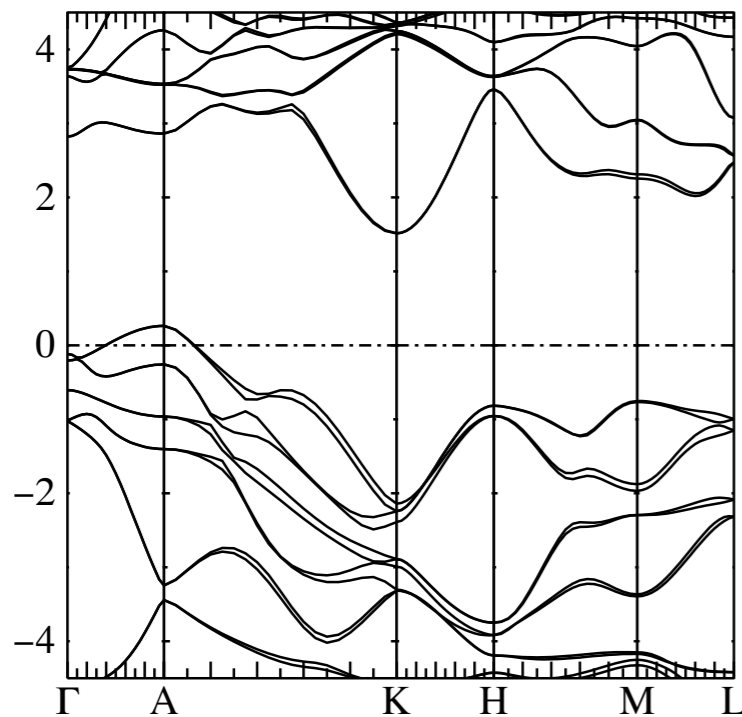
$$H = H_b + H_{so} + H_{ex}$$

changed to

$$H = H_{be} + H_{so}$$

detail of the VB - effect of spin-orbit (SO) int.  
(zero order eff.)

$\vec{m}_{1,2} \parallel \hat{z}$

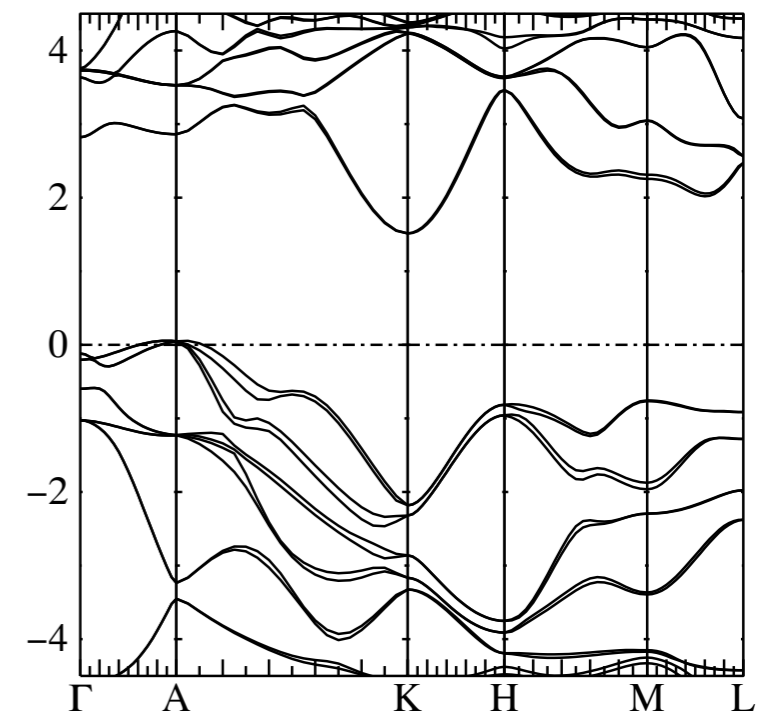


$$H_{so} = \lambda \vec{S} \cdot \vec{L}$$

$$H_{so} = \begin{pmatrix} 0 & i\lambda \\ -i\lambda & 0 \end{pmatrix}$$

$$H_{so} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\vec{m}_{1,2} \parallel \hat{x}$



detail of the VB - effect of spin-orbit (SO) int.  
(zero order eff.)

$$\vec{m}_{1,2} \parallel \hat{z}$$

$$H_{be} = \begin{pmatrix} ak_x^2 & 0 & 0 \\ 0 & bk_x^2 & 0 \\ 0 & 0 & \epsilon_z \end{pmatrix}$$

$$\vec{m}_{1,2} \parallel \hat{x}$$

$$H_{so} = \begin{pmatrix} 0 & i\lambda & 0 \\ -i\lambda & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$H_{so} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & i\lambda \\ 0 & -i\lambda & 0 \end{pmatrix}$$

$$\pm\lambda, \epsilon_z$$

$$0, \lambda^2/\epsilon_z, \epsilon_z - \lambda^2/\epsilon_z$$

$$\sim 1 \text{ eV}$$

$$\sim 1 \text{ meV}$$

$$H_{so} = \Delta_z S_z L_z + \Delta_{xy} (S_x L_x + S_y L_y)$$

$$2\Delta_z$$

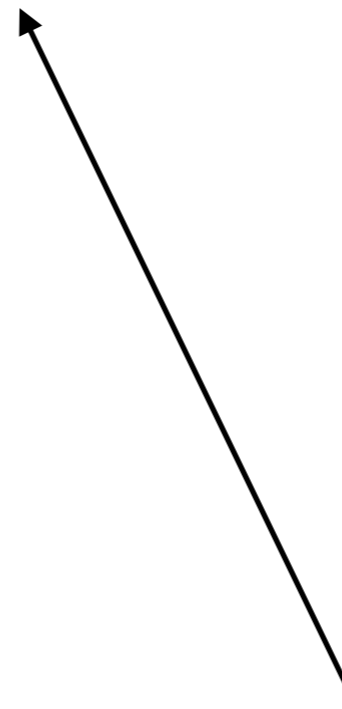
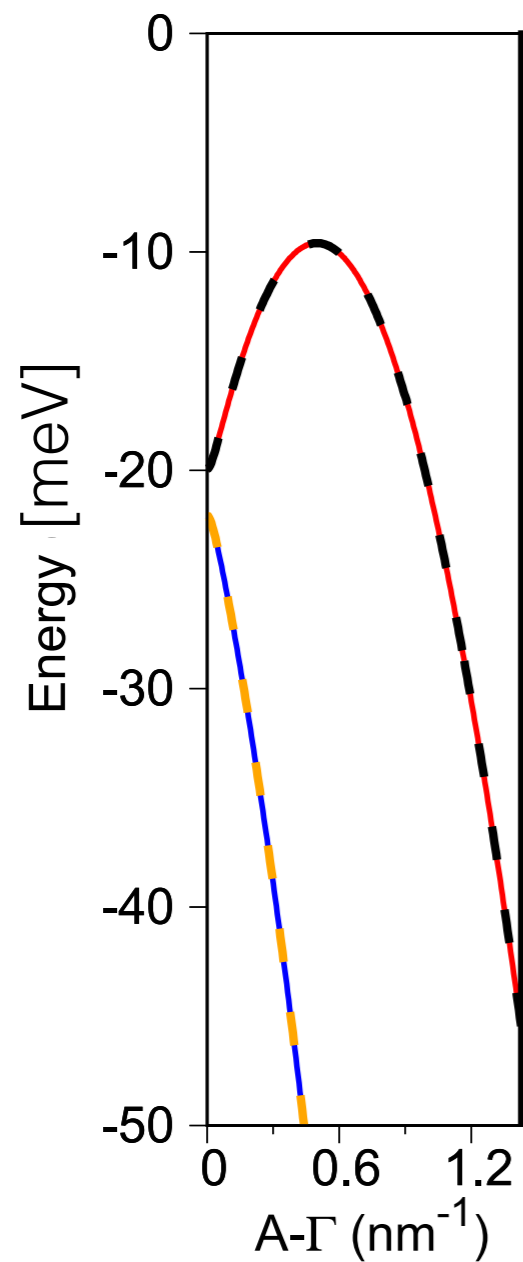
$$\Delta_{xy}^2/\epsilon_z$$

$$\Delta_z \approx 0.27 \text{ eV}$$

$$\Delta_{xy} \approx 0.06 \text{ eV}$$

detail of the VB - effect of spin-orbit (SO) int.  
(next order eff.?)

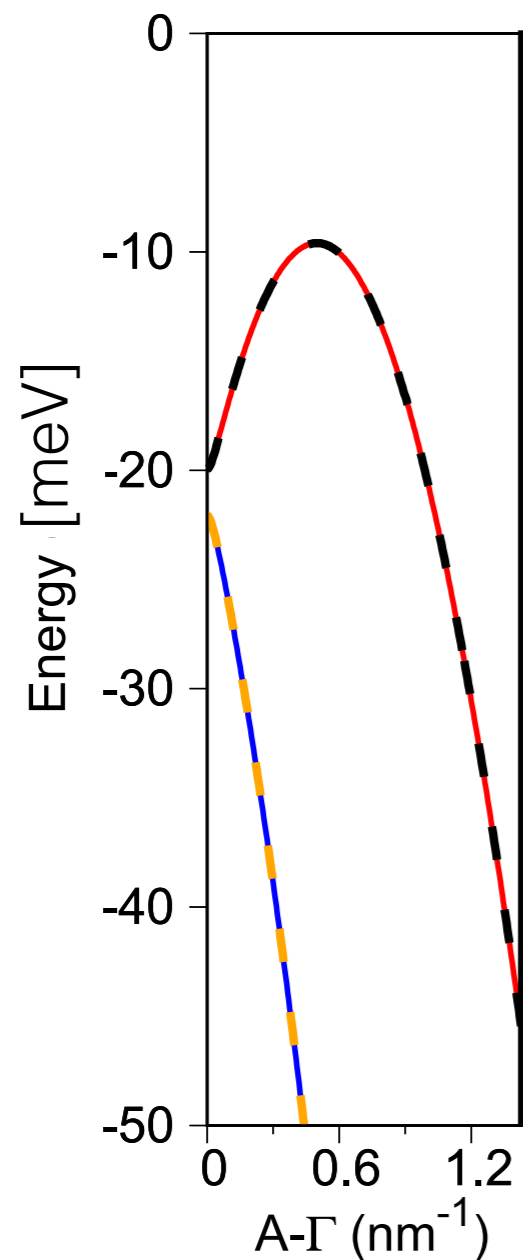
splitting along  
 $A - \Gamma$  line



similar to Rashba?

detail of the VB - effect of spin-orbit (SO) int.  
(first order eff.)

splitting along  
 $A - \Gamma$  line



block structure

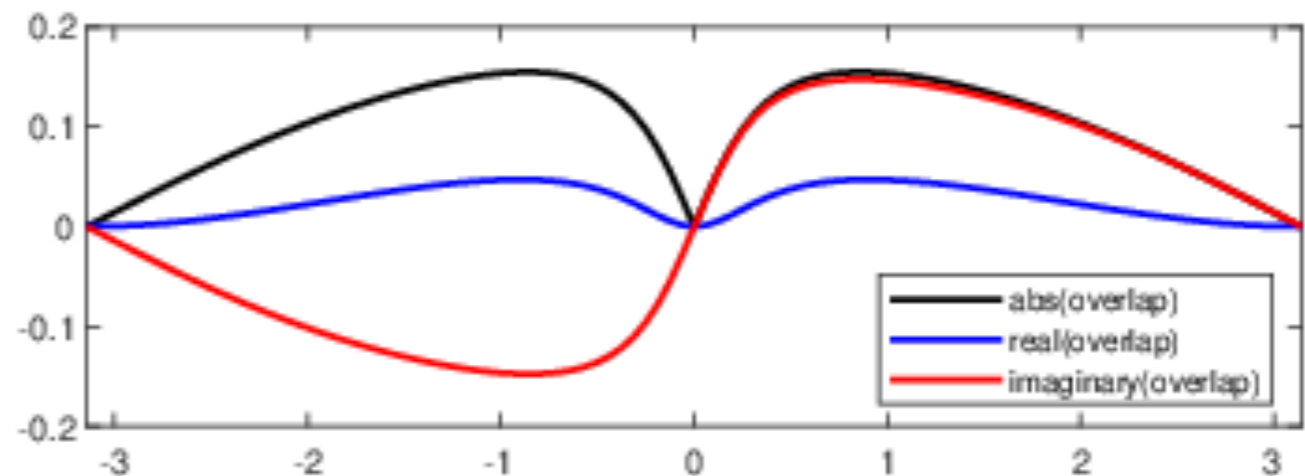
$$(|a\rangle + |b\rangle) \otimes |\uparrow\rangle$$

$$(|a\rangle - |b\rangle) \otimes |\downarrow\rangle$$

$$\begin{pmatrix} ak_x^2 + bk_y^2 + ck_z^2 & x & & \\ x & ak_y^2 + bk_x^2 + ck_z^2 & & \\ & & ak_x^2 + bk_y^2 + ck_z^2 & x \\ & & x & ak_y^2 + bk_x^2 + ck_z^2 \end{pmatrix}$$

$$\begin{pmatrix} & x & & it\Delta_z \\ x & & -it\Delta_z & \\ -it\Delta_z & it\Delta_z & & x \\ & & x & \end{pmatrix}$$

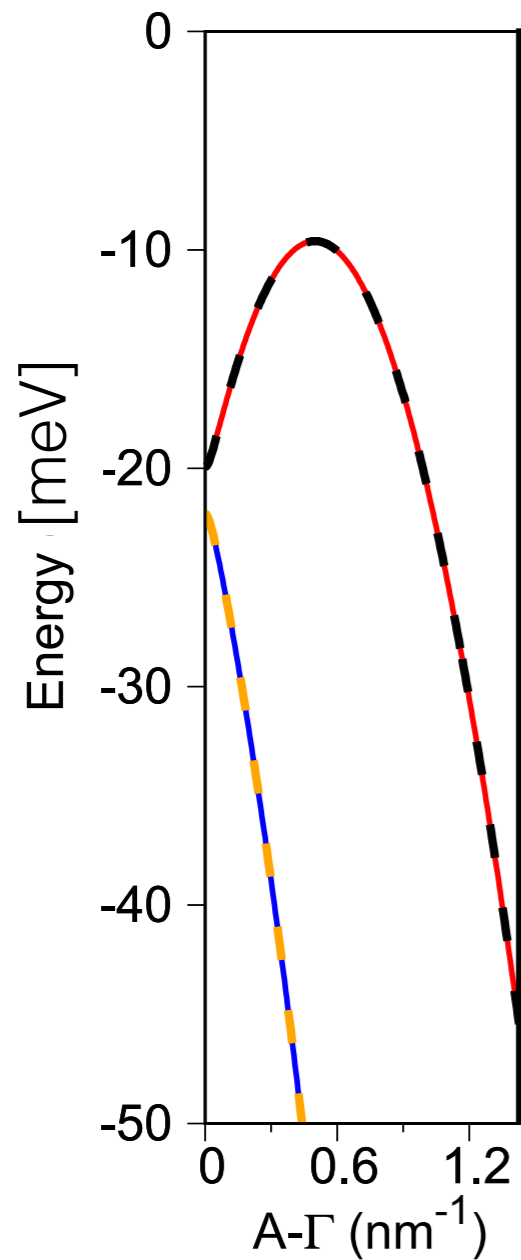
overlaps in the toy model





detail of the VB - effect of spin-orbit (SO) int.  
(first order eff.)

splitting along  
A -  $\Gamma$  line



state 1

$$(|a\rangle + |b\rangle) \otimes |\uparrow\rangle$$

state 2

$$(|a\rangle - |b\rangle) \otimes |\downarrow\rangle$$

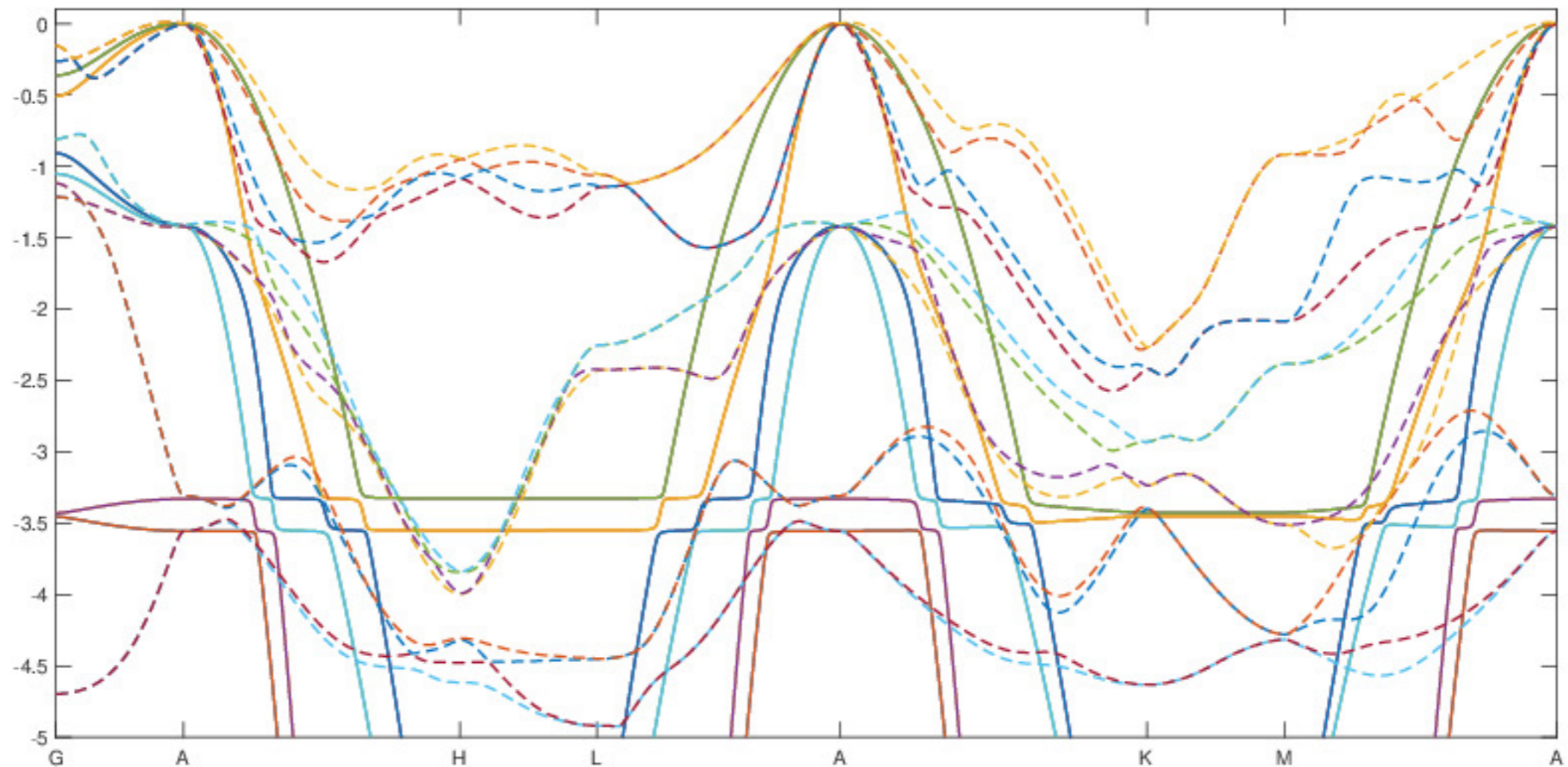
$$H_{be} = \begin{pmatrix} ak_x^2 + bk_y^2 + ck_z^2 & x & 0 & 0 & 0 & 0 \\ x & bk_x^2 + ak_x^2 + ck_z^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \epsilon_z & 0 & 0 \\ 0 & 0 & 0 & 0 & ak_x^2 + bk_y^2 + ck_z^2 & x \\ 0 & 0 & 0 & 0 & x & bk_x^2 + ak_x^2 + ck_z^2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\vec{m}_{1,2} \parallel \hat{x}$$

$$H_{so} = \begin{pmatrix} 0 & 0 & -i\Delta_{xy} & -it\Delta_z & 0 & t\Delta_{xy} \\ 0 & i\Delta_{xy} & -i\Delta_{xy} & -it\Delta_z & 0 & 0 \\ -it\Delta_z & 0 & -t\Delta_{xy} & -t\Delta_{xy} & 0 & 0 \\ t\Delta_{xy} & 0 & 0 & -t\Delta_{xy} & 0 & 0 \\ 0 & it\Delta_z & 0 & 0 & -i\Delta_{xy} & i\Delta_{xy} \\ 0 & 0 & 0 & 0 & i\Delta_{xy} & -i\Delta_{xy} \end{pmatrix}$$

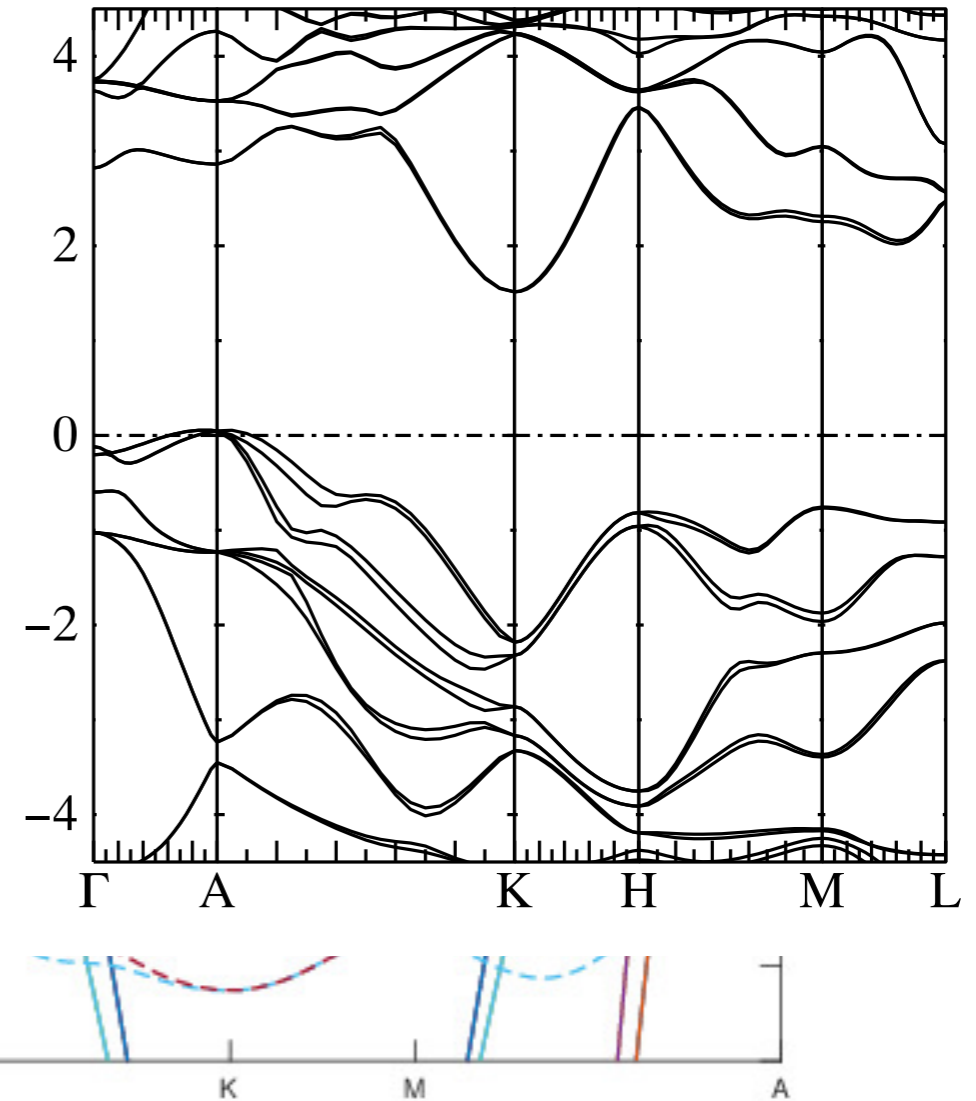
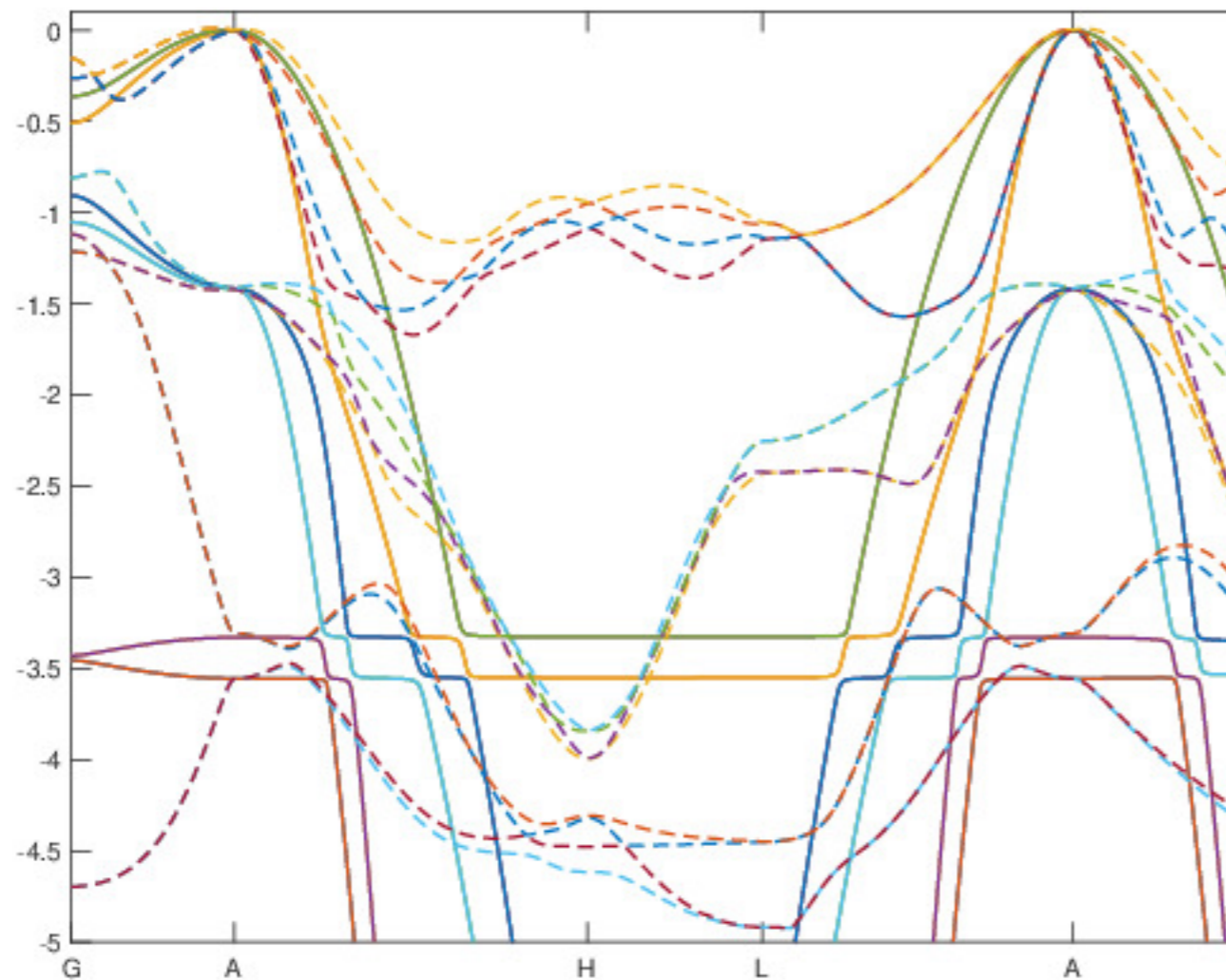
$$t = \langle 1|2\rangle \propto k_z$$

# More realistic toy model: 12 bands



- based on toy model (4 atoms)
- downfolded to two Te atoms
- 3 orbitals (x,y,z)
- 2 spins, incl. SO

# More realistic toy model: 12 bands

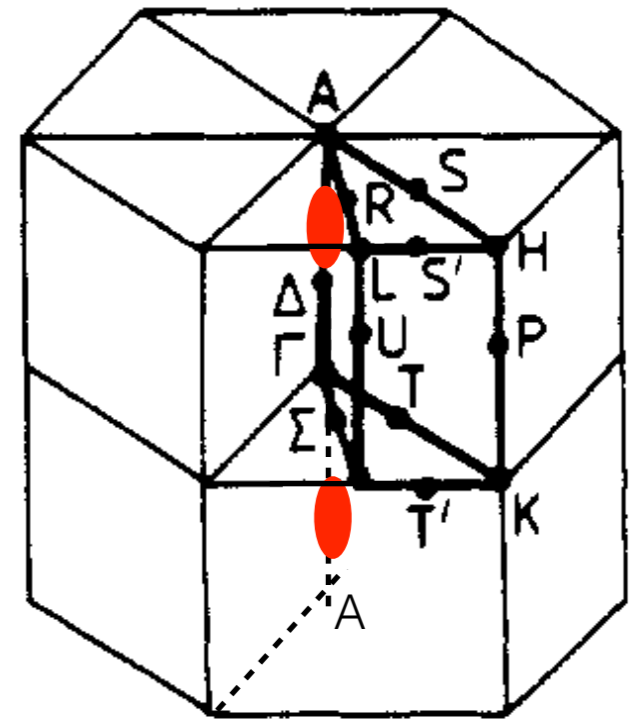
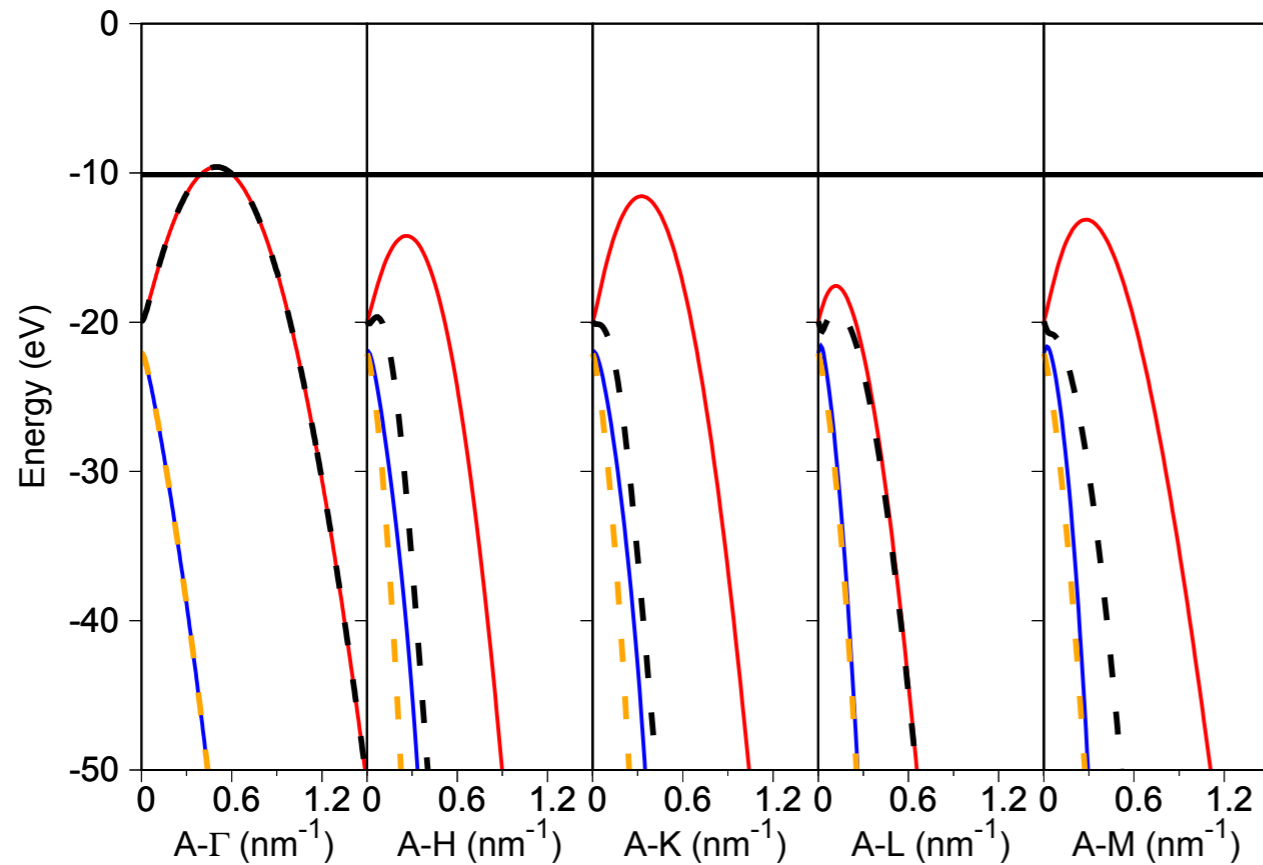


- based on toy model (4 atoms)
- downfolded to two Te atoms

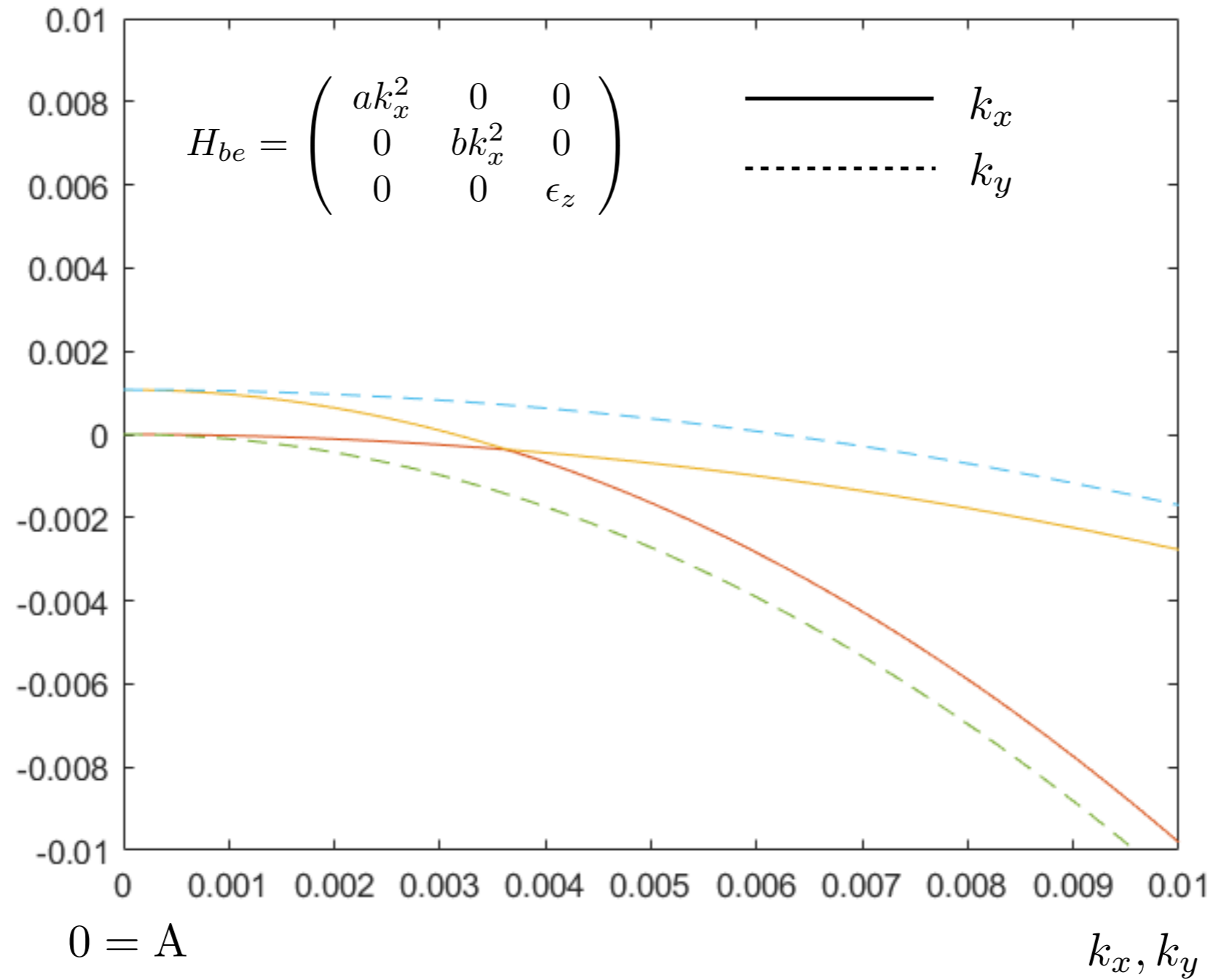
- 3 orbitals (x,y,z)
- 2 spins, incl. SO

detail of the VB - effect of spin-orbit (SO) int.  
(first order eff.)

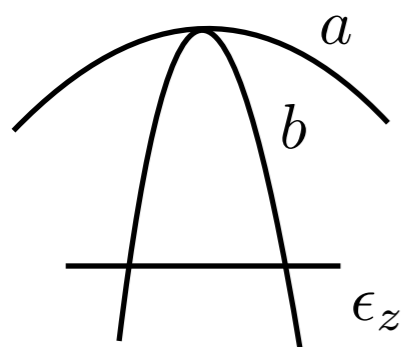
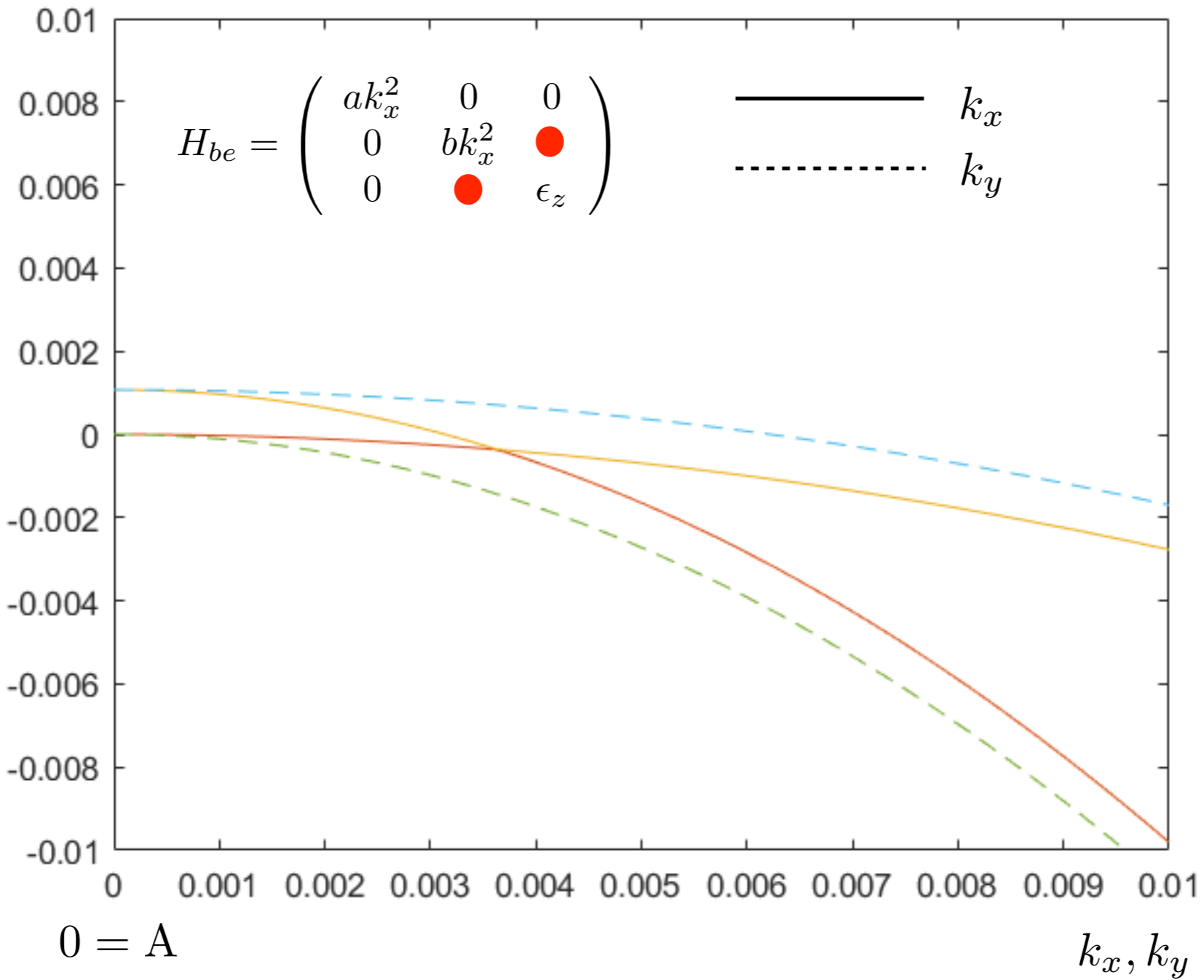
$$H_{be} = \frac{\hbar^2}{2m_0} \begin{pmatrix} ak_x^2 + bk_y^2 + ck_z^2 & (a-b)k_xk_y \\ (a-b)k_xk_y & ak_y^2 + bk_x^2 + ck_z^2 \end{pmatrix}$$



# anisotropic bands (due to exchange+SO)



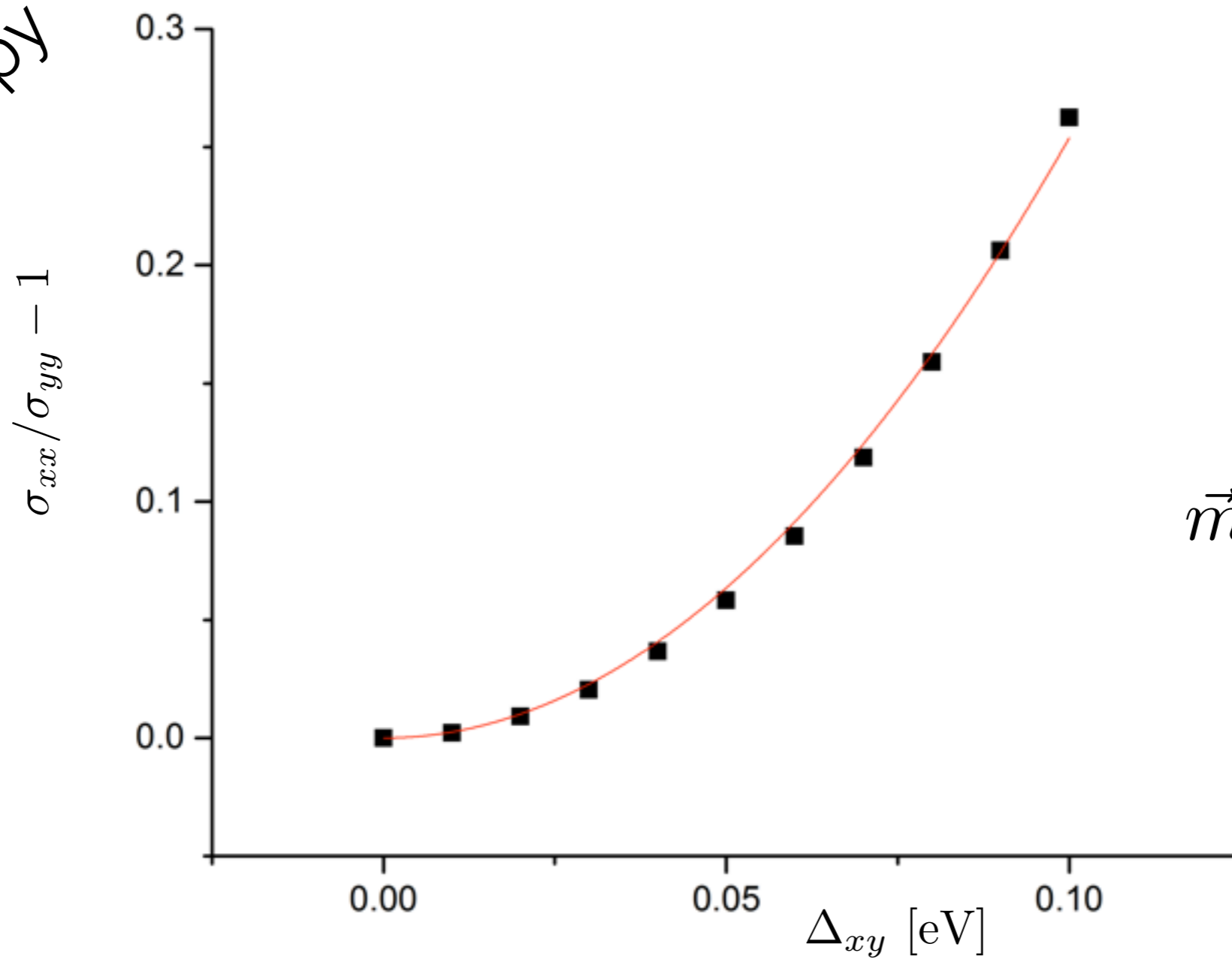
# anisotropic bands (due to exchange+SO)



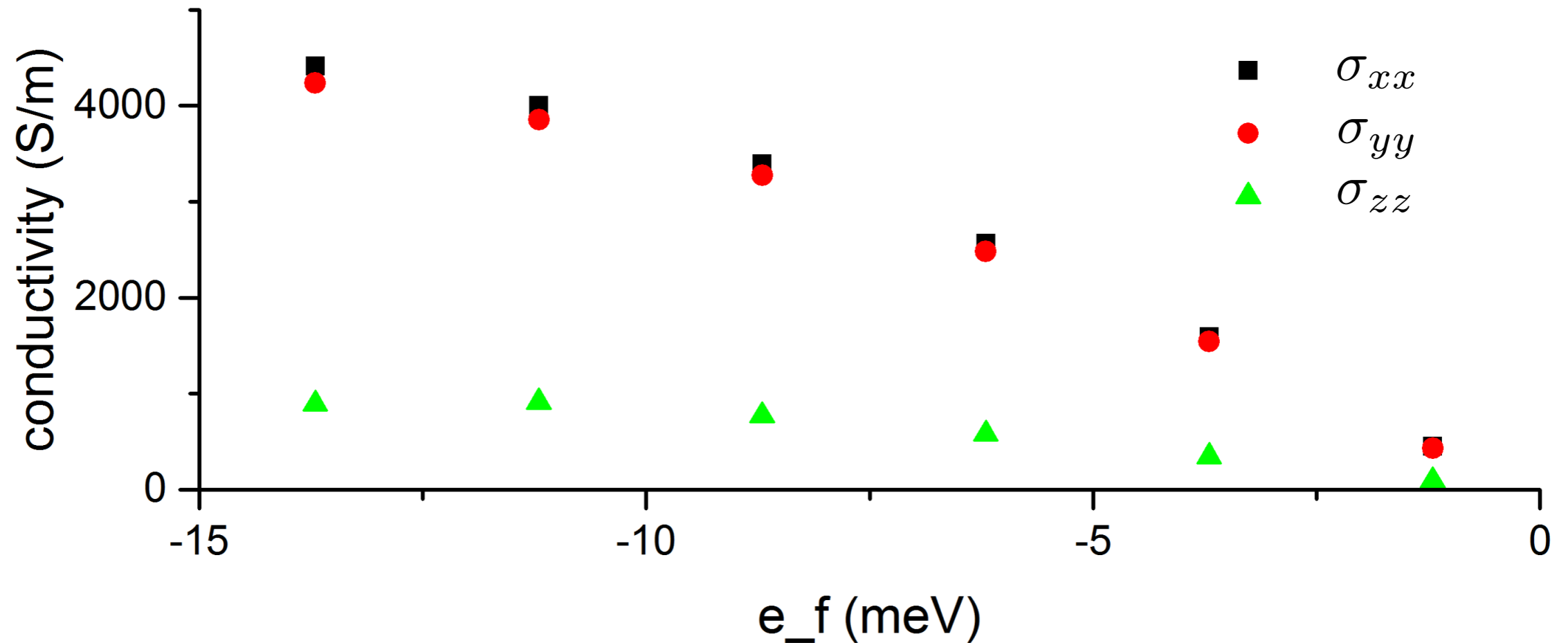
conductivity: Boltzmann approach

$$\sigma_{xx} = e^2 \tau \int \frac{d^3 k}{(8\pi^2)} v_x^2 \delta(E_k - E_F)$$

anisotropy



# conductivity: Boltzmann approach



$$\Gamma_{n,\mathbf{k}} = \frac{2\pi}{\hbar} N_{\text{Mn}} \times \sum_{n'} \int \frac{d^3k'}{(2\pi)^3} |M_{nn'}^{\mathbf{k}\mathbf{k}'}|^2 \delta[E_n(\mathbf{k}) - E_{n'}(\mathbf{k}')] \times (1 - \cos \theta_{\mathbf{v}\mathbf{v}'}),$$

$$\tau = \hbar / 2\Gamma$$

transport relax. time

$$M^C = V(|\mathbf{k} - \mathbf{k}'|)\mathbb{1}, \quad V(q) = -\frac{e^2}{\epsilon} \frac{1}{q^2 + q_{TF}^2}$$

Thomas-Fermi screening

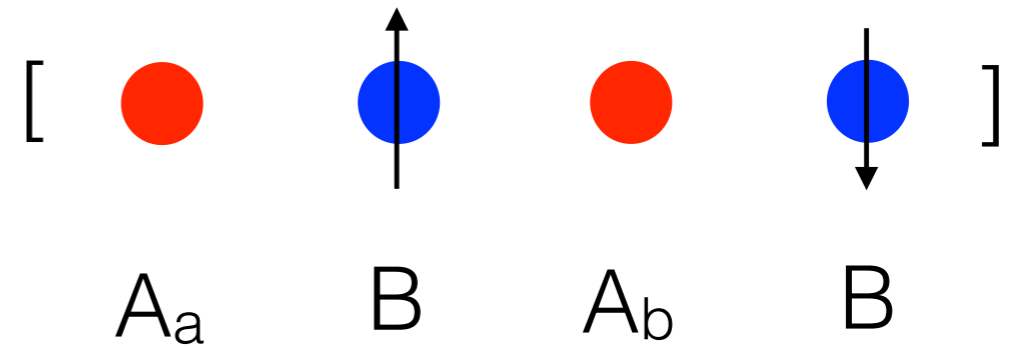
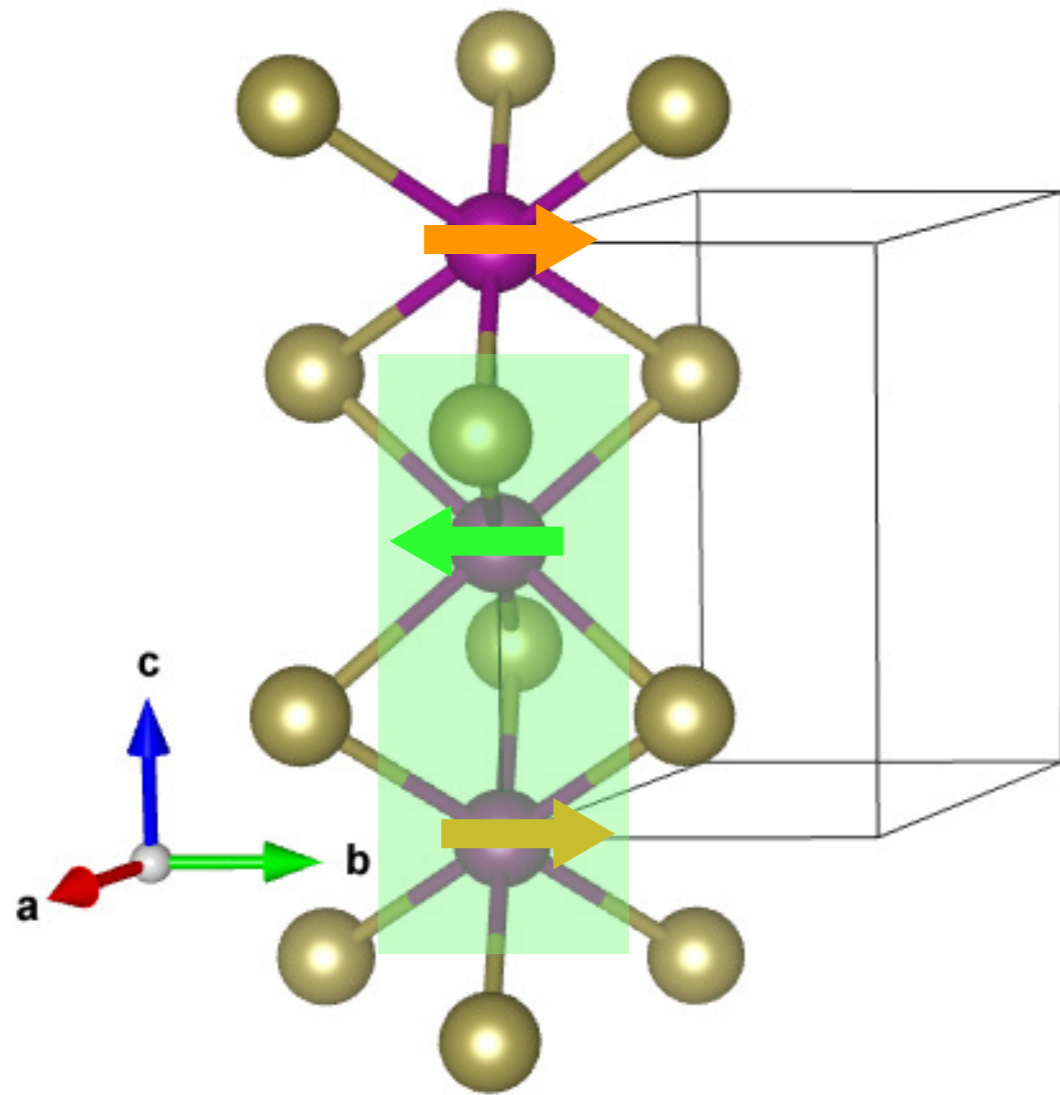


# Conclusions

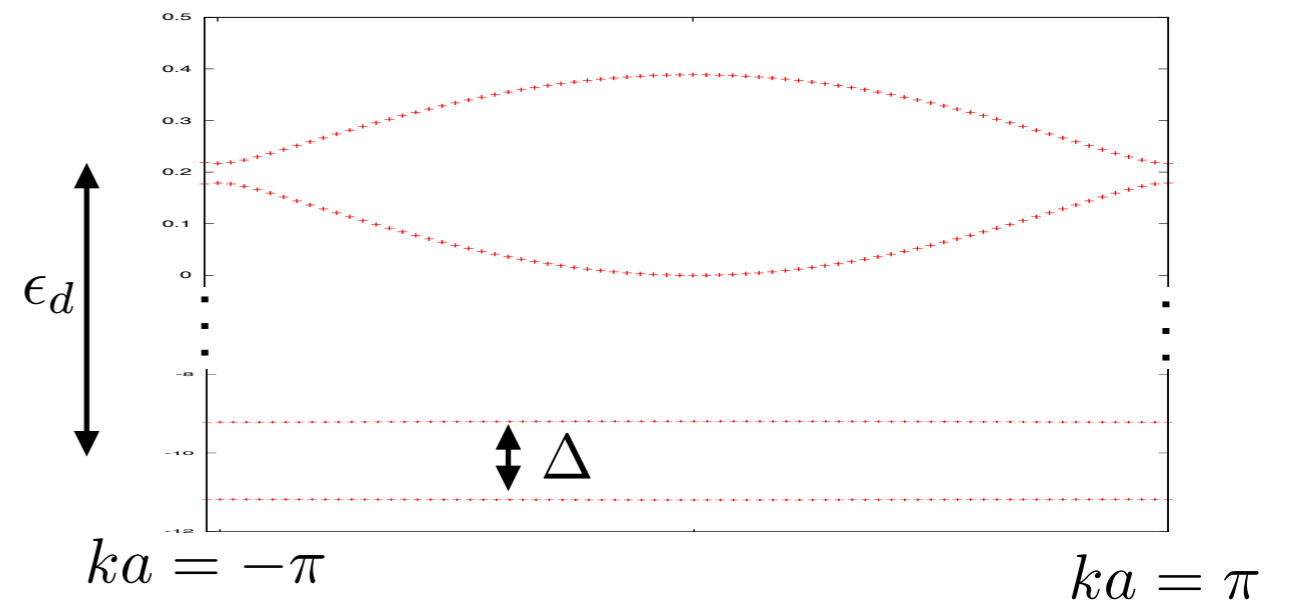
- competing VB maxima - to be resolved by advanced ab initio
- effective model components: band, SO, exchange (band+exchange go together)
- complicated structure in the vicinity of A-point
- conductivity can easily be evaluated, scattering treated e.g. by Fermi golden rule
- some symmetry-breaking terms still to be identified



# how to deal with exchange - a toy model



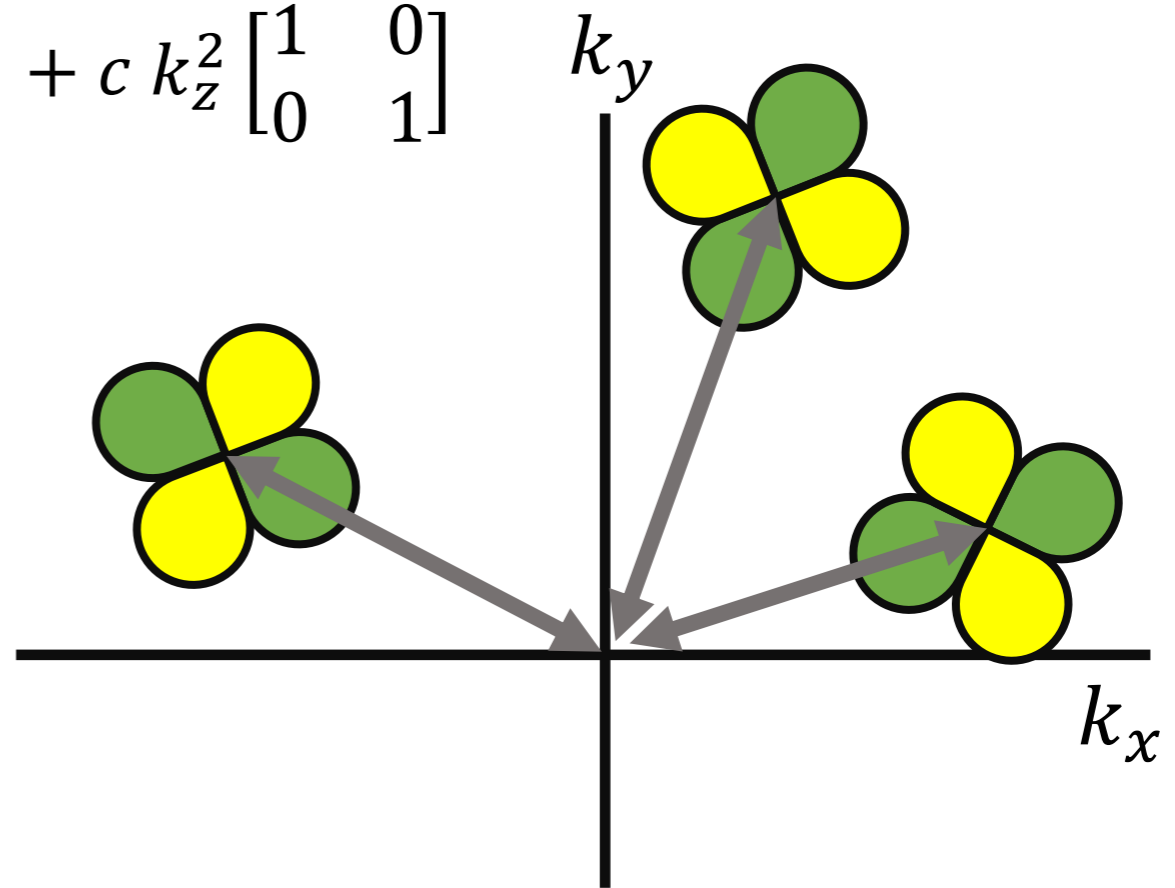
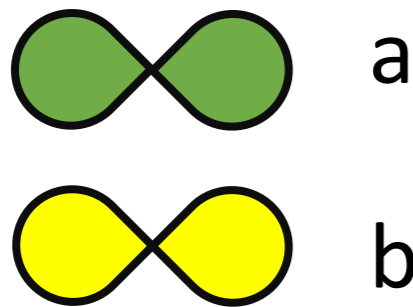
$$H = \begin{pmatrix} 0 & t & 0 & te^{-ika} \\ t & \epsilon_d + \Delta & t & 0 \\ 0 & t & 0 & t \\ te^{ika} & 0 & t & \epsilon_d - \Delta \end{pmatrix}$$



# Crystalline Hamiltonian

- Lowest order expansion

- $$H = \begin{bmatrix} a k_x^2 + b k_y^2 & (a - b)k_x k_y \\ (a - b)k_x k_y & b k_x^2 + a k_y^2 \end{bmatrix} + c k_z^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



What is the basis?

$p_x, p_y$  orbitals

# AMR: crystalline and non-crystalline components

$$\Delta\rho_L/\rho_{av} = C_I \cos 2\phi + C_{I,C} \cos(2\phi + 4\theta) + C_C \cos(4\phi + 4\theta) + C_U \cos(2\phi + 2\theta)$$

$$\Delta\rho_T/\rho_{av} = C_I \sin 2\phi - C_{I,C} \sin(2\phi + 4\theta)$$

de Ranieri et al., NJP '08

