Nodal correlations and the disorder-driven collapse of the Fractional Quantum Hall regime

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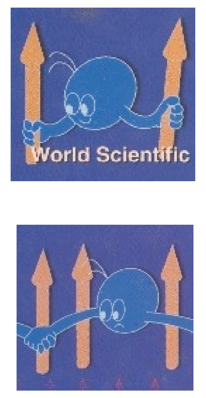
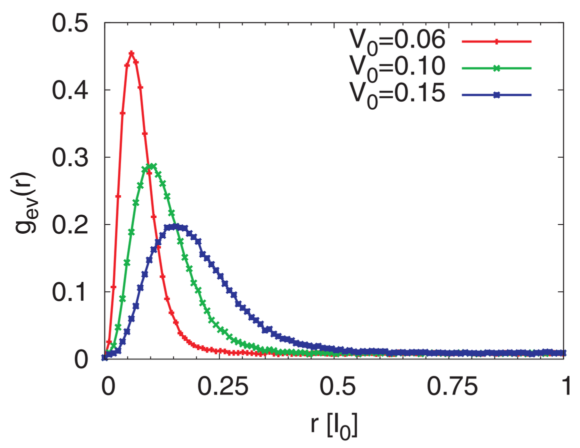
How to dissociate an electron-vortex complex?



- focus on filling factor 1/3, hard-core interaction
- use disorder (inhomogeneous system)
- watch for electron - vortex correlations



... in detail:

- ground state in a homogeneous system: Laughlin wavefunction

$$\Psi(z_1,\ldots,z_N)=\prod_{i=1}^{N}e^{-|z_i|/4}\prod_{j\neq i}(z_i-z_j)^3$$

- zeros vortices flux quanta
- fix all but one particle - there will be triple zeros at $z_2, z_3, \ldots, z_N$: one zero is mandatory (Pauli exclusion principle), the other two can be interpreted as attached flux quanta, dubbed composite fermions
- we are interested in the destruction of those correlations

Possible sources:

inhomogeneities or deviation from hard-core interaction

Types of vortices:

Pauli / center-of-mass / correlation vortices

Procedure:

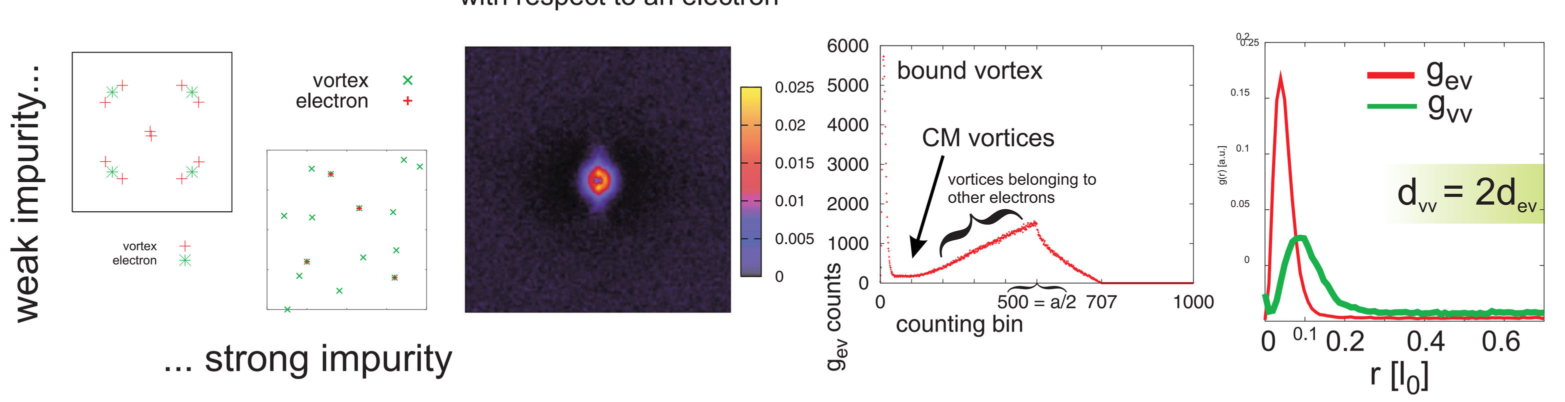
- diagonalize the many-body Hamiltonian ground state $(z_1,z_2,\ldots)$
- create a random distribution of N-1 electrons $(z_2,z_3,\ldots,z_N)$ using a weighted Monte Carlo method, take the reduced wave function $(z) = (z,z_2,\ldots,z_N)$
- find the zeroes Z_i of $|(z)|$ (i=1,...,3N)
- check: calculate vorticity of (z) in each Z_i ($d \ln (z)/d z$ on a small loop around Z_i), verify that it is (a multiple of) 2
- remove the Pauli vortices (those exactly at $z_2,\ldots,z_n$)
- count the remaining vortices using:

$$g_{ev}(r)=\frac{1}{N_{MC}}\sum_{\text{MC steps}}\sum_{i=2}^{N}\sum_{\substack{j=1\\ \text{excl. Pauli}}}^{3N}\delta(r-|Z_j-z_i|)\qquad g_{vv}(r)=\frac{1}{N_{MC}}\sum_{\text{MC steps}}\sum_{\substack{j,j'=1\\ \text{excl. Pauli}}}^{3N}\delta(r-|Z_j-Z_{j'}|)$$

- repeat to get a statistical sample in the sum above

The procedure in figures

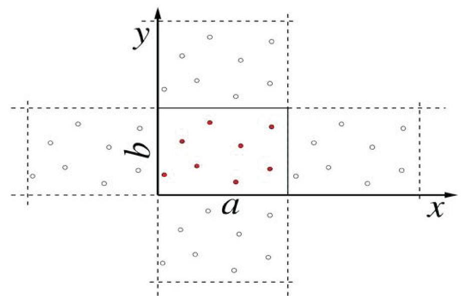
Electrons and vortices — Statistical vortex position with respect to an electron — Finite geometry effects — Focus: bound vortices



The Hamiltonian

$$H=\frac{e^2}{4\pi\varepsilon\ell_0}\sum_{i<j}\frac{1}{|\vec r_i-\vec r_j|/\ell_0}+\sum_i V(\vec r_i)$$

- Coulomb replaced by hard-core repulsion
- constraint to the lowest Landau level (no mixing to higher LLs)
- torus = square, periodic boundary conditions
- zero 2DEG width



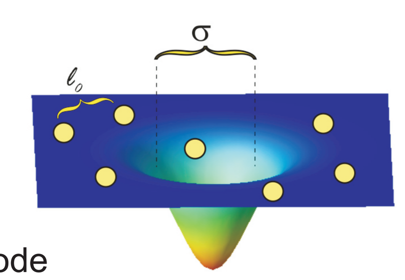
trivia:

- magnetic length $\ell_0=\sqrt{\hbar/eB}$
- filling factor: $\frac{\#\text{ of electrons}}{\#\text{ of mag. flux quanta}}$

$$\nu=\frac{n}{eB/h}=\frac{nA}{AB/(h/e)}$$

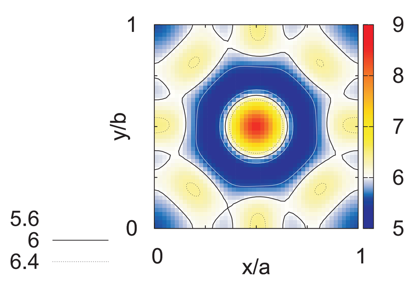
- energy scales: "enu"
  Coulomb units: $e^2/(4\pi\varepsilon\ell_0)\approx 51\,\mathrm{K}\sqrt{B\,[\mathrm{T}]}$
  cyclotron energy: $\hbar\omega\approx 20\,\mathrm{K}\cdot B\,[\mathrm{T}]$

Single impurity, $V(z)=V_0\exp\left[-\frac{(z-z_0)^2}{\sigma^2}\right]$

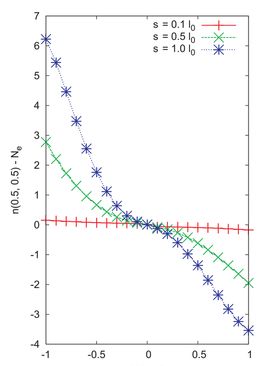


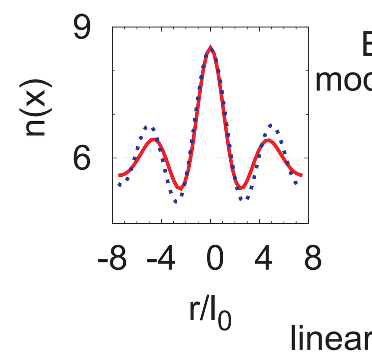
other measures for the effect of disorder:

... change in the density — ... depending on impurity strength — single mode approximation:



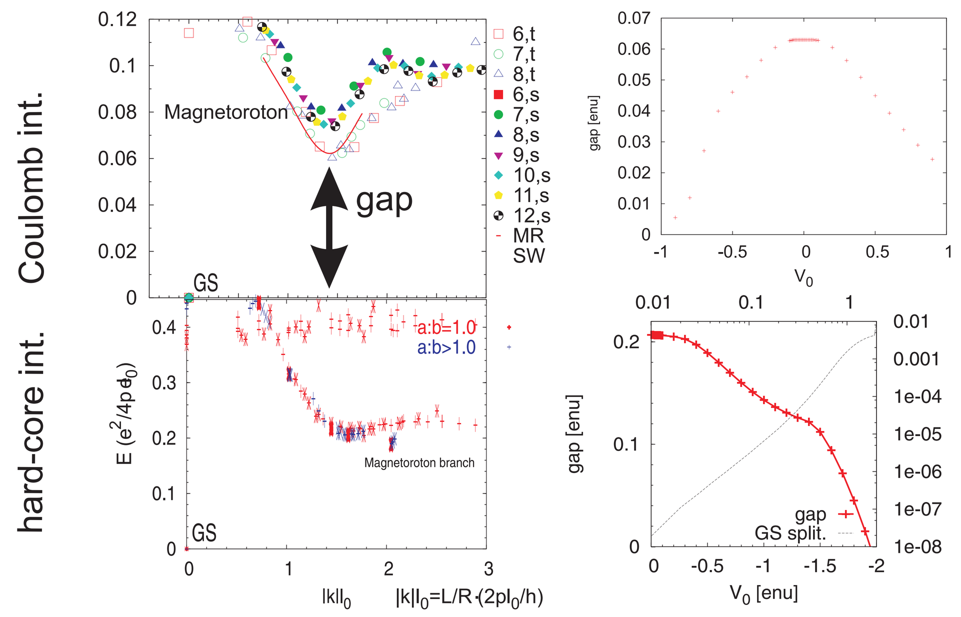
$$n(\vec r)=\langle\Psi|\sum_{i=1}^{N}\delta(\vec r_i-\vec r)|\Psi\rangle$$





linear response model: $n(r)$ $J(kr), k$ MR min.

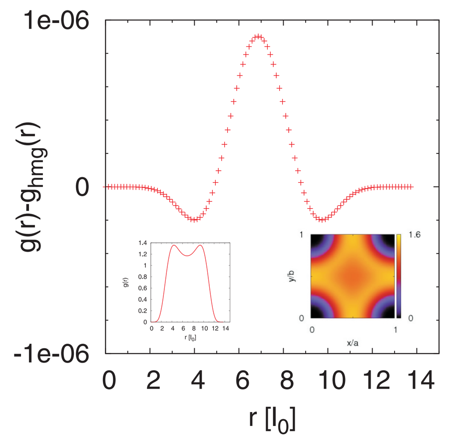
... gap reduction



- density does respond to disorder but this does not necessarily imply collapse of the FQHE
- gaps are reduced by disorder, however only by ≪ V_0 [details in Ref. 1]
- electron-electron correlations change

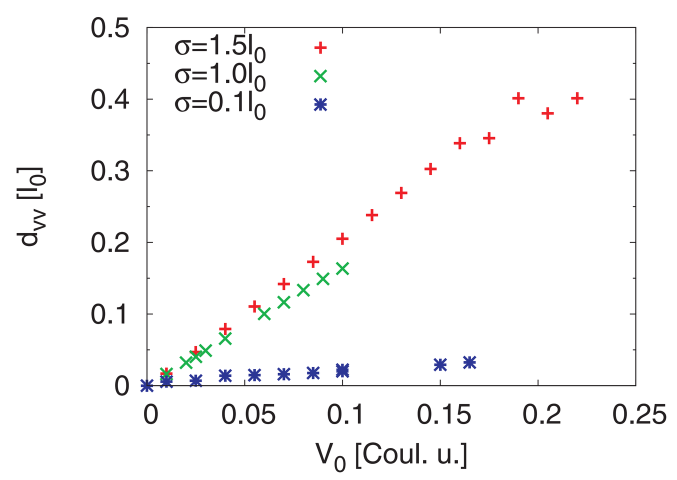
... electron-electron correlation function:

$$g(\vec r)=\langle\Psi|\sum_{i=1}^{N}\delta((\vec r_i-\vec r_j)-\vec r)|\Psi\rangle$$



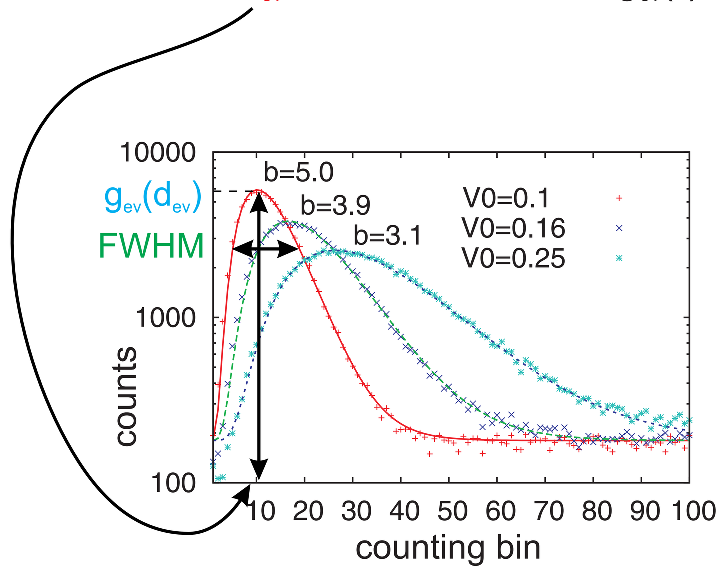
Bound vortex positions:

detachment of electron-vortex complex under increasing V_0



- electron-vortex and vortex-vortex distance proportional to V_0
- l_0 (- impurity) : bound quasielectron, other complexes electron-vortex undisturbed
- l_0 : decaying dependence

Position of d_{ev} - the maximum of $g_{ev}(r)$. Is there any other useful information?



within a range of V_0, good quality fits by

$$g_{ev}(r)=c_0+a\cdot x^8e^{-b\sqrt{x}}$$

slightly less accurate fits possible by

$$g_{ev}(r)=c_0+a\cdot x^4e^{-bx}$$

this implies that for any V0:

d_{ev} FWHM $1/g_{ev}(d_{ev})$

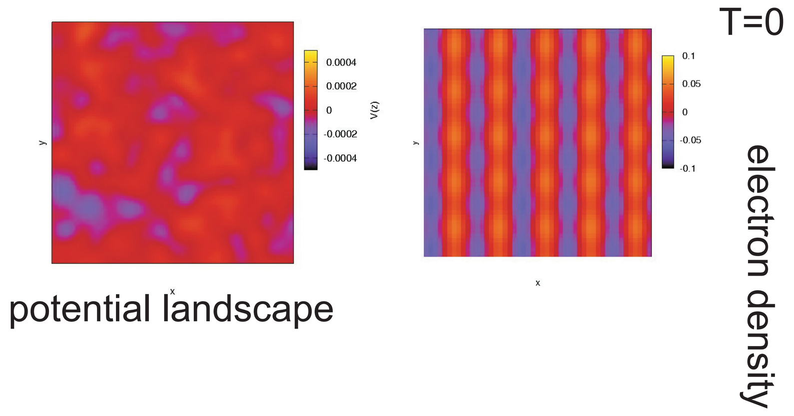
in particular:
d_{ev}=4/b, FWHM=1.19d_{ev}, w=0.781/d_{ev}

Multiple impurities

$$V(z)=\sum_{i=1}^{N_{imp}}V_i\exp\left[-\frac{(z-z_i)^2}{\sigma^2}\right]$$

"Universal properties of a disordered system presumably depend only on a few statistical properties of the Hamiltonian like the correlation function." [B. Huckestein]

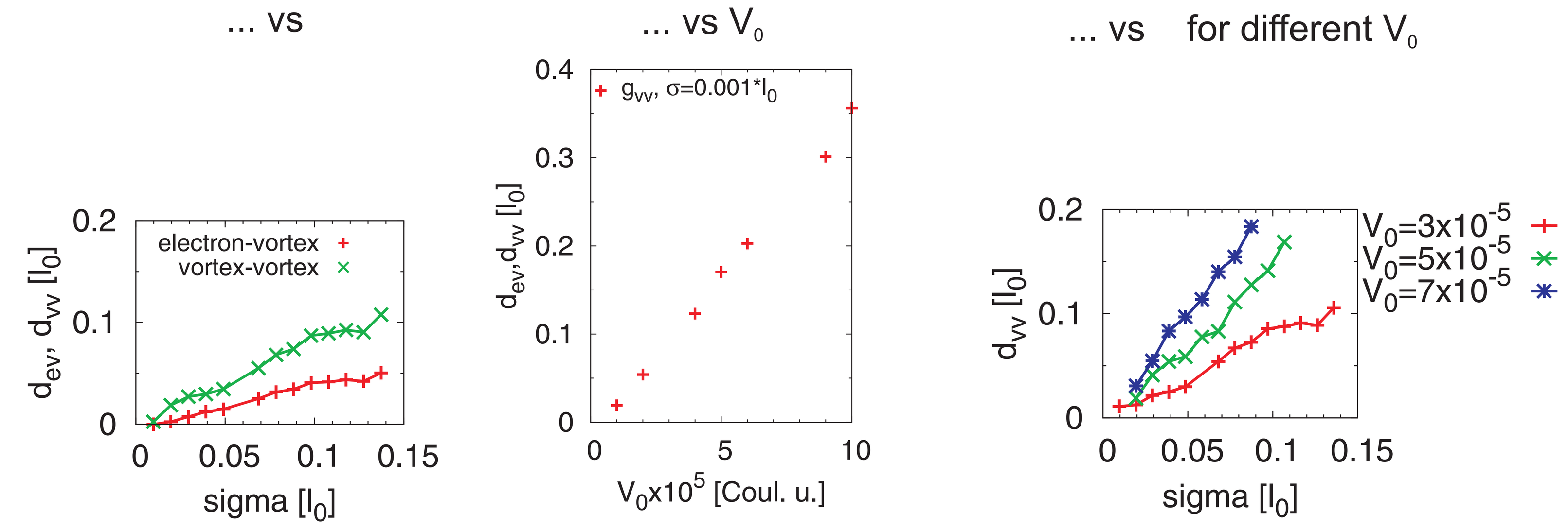
$$\overline{V(\vec r)V(\vec r')}\propto\frac{1}{2\pi\sigma^2}\exp(-|\vec r-\vec r'|^2/2\sigma^2)$$



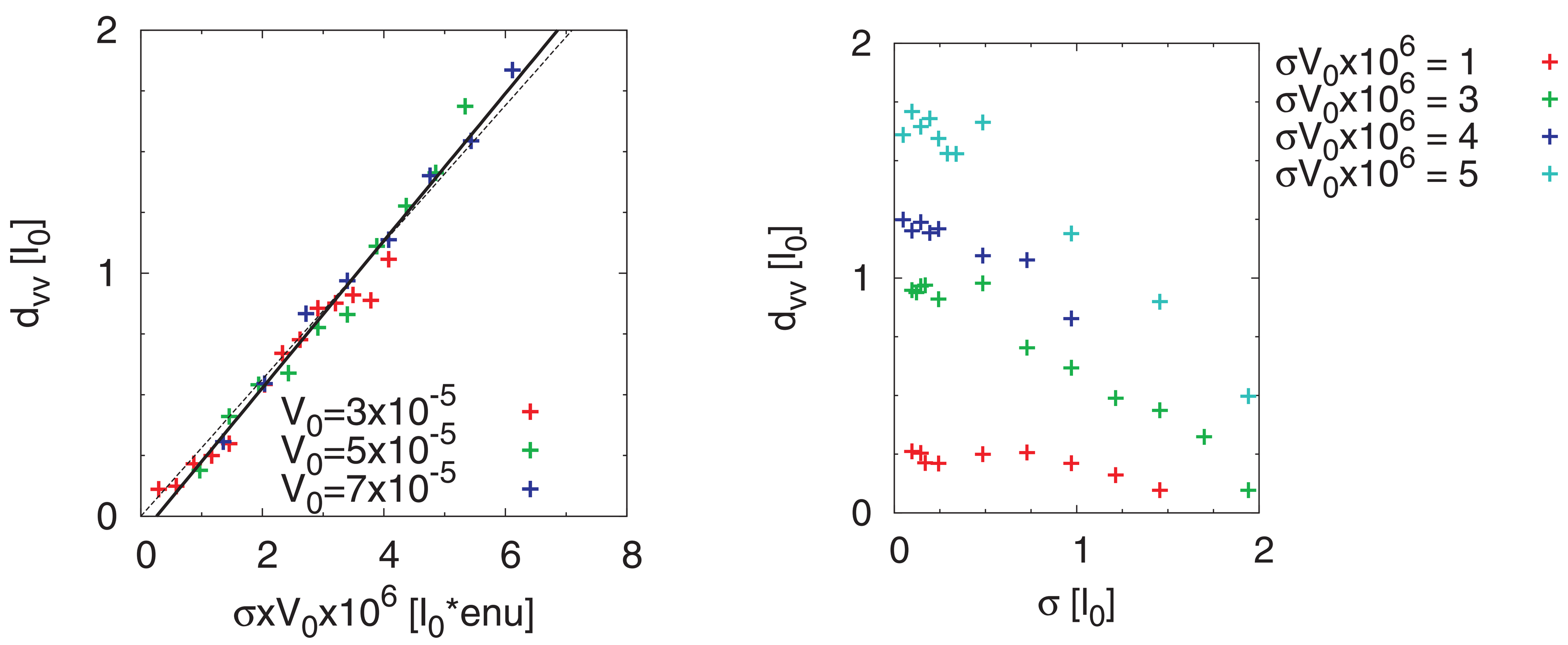
potential landscape

Finite correlation length = sigma

Electron-vortex and vortex-vortex distances...



Electron-vortex distance depends only on V_0 ... as long as << l_0



References: [1] K. Graham et al., Phys. Rev. B 67, 235302 (2003)
[2] Gun Sang Jeon et al., Phys. Rev. B 72, 035304 (2005)
[3] B. Huckestein, Rev. Mod. Phys. 67, 357 (1995)
[4] K. Vyborny et al., Acta Phys. Pol. A 112, 249 (2007)
[5] K. Vyborny et al., cond-mat/0703109 (unpublished, 2007)
[6] K. Vyborny, Annalen der Physik (Leipzig) 16, 87 [2] (2007).

Conclusions:

- an electron is surrounded by two vortices situated opposite to each other, in the =1/3 FQHE state
- electron-vortex distance scales linearly with the amplitude of impurity potential (=sensitive probe of the disorder effects)
- under disorder which hardly changes the gap the vortex-electron distance responds sensitively
- even if vortices are effectively detached, the FQH gap may remain in place
- a promising method to study the weakening of Laughlin type correlations in a finite system is proposed