Hybrid skew scattering regime of the anomalous Hall effect in Rashba systems: unifying Keldysh, Boltzmann, and Kubo formalisms.

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(Dated: June 24, 2008)

We present the analytical description of the anomalous Hall effect (AHE) in a 2DEG ferromagnet within the Keldysh formalism. These results unify the three linear response approaches to AHE and close the debate on previous discrepancies. We are able to identify a new extrinsic AHE regime dominated by a hybrid skew scattering mechanism. This new contribution is inversely proportional to the impurity concentration, resembling the normal skew scattering, *but* independent of the impurity-strength, resembling the side-jump mechanism. Within the Kubo formalism this regime is captured by higher order diagrams which, although weak, can dominate when both subbands are occupied; this regime can be detected by variable remote doping experiments.

PACS numbers: 72.15.Eb, 72.20.Dp, 72.20.My, 72.25.-b

The anomalous Hall effect (AHE) has been a subject of fundamental research in condensed matter physics for many decades. The anomalous Hall resistivity ρ_{xy} describes the transverse voltage with respect to the transport direction and depends on the spontaneous magnetization M along the z direction. The origin of the AHE lies in the intrinsic band structure properties [1], and extrinsic spin-asymmetric scattering such as skew-scattering [2] and side-jump scattering [3].

Even though the AHE has been studied for a long time [4], it still remains a controversial theoretical subject due to the difficulty to obtain agreement between the different linear response calculations within equivalent systems [5, 6, 7, 8, 9, 10, 11]. Recently, some consensus has been reached between the diagrammatic Kubo formalism [10] and the Boltzmann approach [9]. Application of the Keldysh formalism to the problem is relatively new [6, 7] and connection to the previous theories is required. Liu *et al.* [6] employ this approach but fail to reproduce the diagrammatic results [10] because the gradient expansion of the collision integral is not taken into account [9]. Onoda *et al.* [7] use the Keldysh technique formulated in a gauge invariant way; however, employment of the non-chiral basis representation lacks transparency and only allows for a numerical solution.

In this Rapid Communication, we derive the kinetic equation that takes into account both the effects of the Berry curvature and the extrinsic effects. We solve the quantum Boltzmann equation analytically in the metallic (weak scattering) regime, finding the Hall current up to zeroth order in the impurity strength. Employing the chiral basis allows us to immediately identify semiclassical contributions [9] such as intrinsic, side-jump and skew-scattering and therefore make a systematic derivation of the Boltzmann semiclassical approach. We also make a full connection to the results of the previous works using the Kubo formalism [8, 9, 10, 12], hence bringing to an end the long standing theoretical debate within the weak scattering regime. In addition, we calculate the important higher order (hybrid) skew-scattering diagrams. In the limit of high density and mobility, this hybrid-skew-scattering contribution dominates in the metallic regime and surprisingly has no dependence on the scattering strength *but* it is inversely proportional to the impurity concentration.

The method presented in the following is general, however, in order to obtain simple analytical results that connect directly with other microscopic linear response calculations [8, 9, 10], we restrict ourselves to 2D Rashba Hamiltonian with additional exchange field h:

$$\hat{H}_R = \vec{\pi}^2/2m + \alpha\vec{\pi}\cdot\hat{\boldsymbol{\sigma}}\times\mathbf{z} - h\hat{\sigma}_z + V(\mathbf{r}), \tag{1}$$

where $\hat{\boldsymbol{\sigma}}$ are Pauli matrices, $\vec{\pi} = \mathbf{k} - e\mathbf{A}$, $\mathbf{A}(t) = -\mathbf{E}t$ describes the external electric field and $V(\mathbf{r})$ the impurities. Here and throughout the text we take $\hbar = c = 1$. We employ a simplified model of impurity scattering, particularly $V(\mathbf{r}) = V_0\sum_i \delta(\mathbf{r}-\mathbf{r}_i)$, where $\mathbf{r}_i$ describes the positions of randomly distributed impurities. We also estimate the spin-orbit coupling component of the disorder potential and show to be important only in the very high density regime [16, 17].

We start by writing the Dyson equation [15]:

$$\begin{pmatrix} [\hat{G}_0^R]^{-1} - \hat{\Sigma}^R & -\hat{\Sigma}^K \\ 0 & [\hat{G}_0^A]^{-1} - \hat{\Sigma}^A \end{pmatrix} \otimes \begin{pmatrix} \hat{G}^R & \hat{G}^K \\ 0 & \hat{G}^A \end{pmatrix} = \check{1}, \tag{2}$$

where R, A, and K stand for retarded, advanced and Keldysh components of the Green's functions and self-energies, and the subscript 0 labels the disorder free system. The symbol $\otimes$ denotes a convolution (in position, time and spin). By considering Eq. (2) and its conjugate, we arrive at the Kinetic equation [15]:

$$\begin{aligned} [\hat{G}_0^R]^{-1} \otimes \hat{G}^< - \hat{G}^< \otimes [\hat{G}_0^A]^{-1} = \hat{\Sigma}^R \otimes \hat{G}^< - \\ \hat{G}^< \otimes \hat{\Sigma}^A + \hat{\Sigma}^< \otimes \hat{G}^A - \hat{G}^R \otimes \hat{\Sigma}^< \end{aligned}, \tag{3}$$

where $\hat{G}^{<}/\hat{\Sigma}^{<} \equiv (\hat{G}^K/\hat{\Sigma}^K + \hat{G}^A/\hat{\Sigma}^A - \hat{G}^R/\hat{\Sigma}^R)/2$. The iterative version of Eq. (2) corresponding to the repeated scattering by impurities is $\check{\Sigma} = \check{\Sigma}_0 \otimes \left[\check{1} + \check{G} \otimes \check{\Sigma}\right]$, where $\check{\Sigma}_0$ is the self-energy from a single scattering event in Keldysh space. The Keldysh component of this equation, gives the relation:

$$\hat{\Sigma}^{<} = \left[1 + \hat{G}^R \otimes \hat{\Sigma}^R\right] \otimes \hat{\Sigma}_0^{<} \otimes \left[1 + \hat{\Sigma}^A \otimes \hat{G}^A\right] + \hat{\Sigma}^R \otimes \hat{G}^{<} \otimes \hat{\Sigma}^A, \tag{4}$$

where for a single scattering event we have $\hat{\Sigma}_0^{<} = 0$. Equations (3) and (4) form a general closed set of equations for $\hat{G}^{<}$ and are solved in the following.

In the presence of slowly varying perturbations, it is useful to perform the Wigner transformation, *viz.* the center-of-mass coordinates ($X = (R,T)$) and the Fourier transform with respect to the relative coordinates ($k = (\mathbf{k}, \omega)$). In such representation, the convolution of two operators is approximated as $\hat{A} \otimes \hat{B} \approx \hat{A}\hat{B} + \frac{i}{2}\left(\partial_X \hat{A} \partial_k \hat{B} - \partial_k \hat{A} \partial_X \hat{B}\right)$, where we use the four vector notations $\partial_X \partial_k = \partial_{\mathbf{R}} \partial_{\mathbf{k}} - \partial_{\widetilde{T}} \partial_\omega$ and $\partial_{\widetilde{T}} = \partial_T + e\mathbf{E}\partial_{\mathbf{k}}$. Applying this to the Kinetic Eq. (3) we obtain:

$$\left[\hat{H}_0, \widehat{G}^{<}\right] + \frac{i}{2}\left\{e\mathbf{E}\hat{\boldsymbol{v}}_0, \partial_\omega \widehat{G}^{<}_{eq}\right\} + ie\mathbf{E}\partial_{\mathbf{k}}\widehat{G}^{<}_{eq} = \hat{\Sigma}^R \widehat{G}^{<} - \widehat{G}^{<}\hat{\Sigma}^A + \hat{\Sigma}^{<}\widehat{G}^A - \widehat{G}^R \hat{\Sigma}^{<} + \frac{i}{2}\left(\left[\hat{\Sigma}^R, \widehat{G}^{<}_{eq}\right]_p - \left[\widehat{G}^{<}_{eq}, \hat{\Sigma}^A\right]_p + \left[\hat{\Sigma}^{<}_{eq}, \widehat{G}^A\right]_p - \left[\widehat{G}^R, \hat{\Sigma}^{<}_{eq}\right]_p\right), \tag{5}$$

where $[\hat{A}, \widehat{B}]_p \equiv (\partial_X \hat{A} \partial_k \widehat{B} - \partial_k \widehat{A} \partial_X \hat{B})$, $\hat{\boldsymbol{v}}_0 = \partial_{\mathbf{k}} \hat{H}_0$, $\widehat{G}^{<}_{eq} = n_F(\widehat{G}^A - \widehat{G}^R)$ and $\hat{\Sigma}^{<}_{eq} = n_F(\hat{\Sigma}^A - \hat{\Sigma}^R)$. In deriving Eq. (5), one retains only the first order terms in electric field $\mathbf{E}$ and use the fact that our system is homogeneous and stationary ($\partial_{\mathbf{R}} \widehat{G}^{<} = 0$, $\partial_T \widehat{G}^{<} = 0$).

To establish the connection with the several mechanisms identified semiclassically when interpreting the AHE, we transform Eq. (5) into the chiral basis in which $\hat{H}_0$ takes the diagonal form $\hat{S}^\dagger \hat{H}_0 \hat{S} = \widehat{1}k^2/2m - \widehat{\sigma}_z \lambda$, $\hat{\boldsymbol{v}} = \hat{S}^\dagger \hat{\boldsymbol{v}}_0 \hat{S}$, and:

$$\hat{S} = \begin{pmatrix} \cos\theta/2 & \sin\theta/2 \\ ie^{i\varphi}\sin\theta/2 & -ie^{i\varphi}\cos\theta/2 \end{pmatrix},$$

where $\lambda = \sqrt{(\alpha k)^2 + h^2}$, $\cos(\theta) = h/\lambda$ and $\tan(\phi) = k_y/k_x$. We first obtain the intrinsic Hall effect by disregarding the collision integral in the r.h.s. of Eq. (5). In the chiral basis, only nondiagonal terms of $\widehat{G}^{<}$ give nonzero contributions to the intrinsic AHE and they are:

$$\begin{aligned} G^{c<}_{+-} &= ieE\left(iv_y^{+-}\partial_\omega\left[n_F(A^+ + A^-)\right]/2 + \left(G^R_+ G^R_- - G^A_+ G^A_-\right)v_y^{+-} n_F\right)/2\lambda \\ G^{c<}_{-+} &= -ieE\left(iv_y^{-+}\partial_\omega\left[n_F(A^+ + A^-)\right]/2 + \left(G^R_+ G^R_- - G^A_+ G^A_-\right)v_y^{-+} n_F\right)/2\lambda, \end{aligned} \tag{6}$$

where $G^{R(A)}_\pm = 1/(\omega - E_\pm + (-)i\Gamma^{(*)}_\pm)$, $\Gamma_\pm = \Gamma \mp \Gamma_z \dfrac{h}{\lambda}$, $E_\pm = k^2/2m \pm \lambda$, $A^\pm = i(G^R_\pm - G^A_\pm)$ and $\hat{G}^{c<}$ is $\hat{G}^{<}$ in the chiral basis (Γ and Γ_z are defined below; however, they don't affect the intrinsic current in the vanishing Γ limit). The Green's function $\widehat{G}^{c<}$ allows us to find the intrinsic Hall current along the x- axis:

$$j_x = -ie\int \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{d\omega}{2\pi}\mathrm{Tr}[\widehat{G}^{c<}\hat{v}_x] = \tag{7}$$

$$-ie^2E\int \frac{d^2\mathbf{k}}{(2\pi)^2}\frac{d\omega}{2\pi} n_F \frac{(v_y^{+-}v_x^{-+} - v_y^{-+}v_x^{+-})(A^+ - A^-)}{4\lambda^2} = E\frac{e^2}{4\pi}\left(1 - \tfrac{h}{\lambda_-} - (1 - \tfrac{h}{\lambda_+})\theta(\omega_F - h)\right), \tag{8}$$

where $\lambda_\pm = \sqrt{(\alpha k_\pm)^2 + h^2}$ and $k^2_\pm = 2m(\omega_F \mp \lambda_\pm)$ describe Fermi vectors for the lower/upper chiral bands.

The intrinsic solution Eq. (6) contains both the contribution at the Fermi level and from the Fermi sea, often referred to as σ^{II}_{xy} conductivity within the Kubo-Streda formalism. Our next aim is to find the contributions that arise due to impurity scattering at the Fermi level. We separate Eq. (5) into two parts, one is proportional to n_F and the other is proportional to $\partial_\omega n_F$. The part proportional to $\partial_\omega n_F$, i.e. the Fermi-surface, is:

$$\left[\hat{H}_0, \widehat{G}^{<}\right] - \frac{\partial_\omega n_F}{2}\left\{e\mathbf{E}\hat{\boldsymbol{v}}_0, \hat{A}\right\} = \hat{\Sigma}^R\widehat{G}^{<} - \widehat{G}^{<}\hat{\Sigma}^A + \hat{\Sigma}^{<}\widehat{G}^A - \widehat{G}^R\hat{\Sigma}^{<} - \frac{\partial_\omega n_F}{2}(\hat{\Gamma}\partial_{\mathbf{k}}\widehat{G}^A + \partial_{\mathbf{k}}\widehat{G}^R\hat{\Gamma} - \hat{A}\partial_{\mathbf{k}}\widehat{\Sigma}^A - \partial_{\mathbf{k}}\widehat{\Sigma}^R\hat{A}), \tag{9}$$

where $\hat{A} = i(\hat{G}^R - \hat{G}^A)$ and $\hat{\Gamma} = i(\hat{\Sigma}^R - \hat{\Sigma}^A)$. Note that $\partial_{\mathbf{k}}\hat{\Sigma}^{R(A)} = 0$ for the simple delta scatterers. We calculate $\hat{\Sigma}^{R(A)<}$ and Green's functions $\widehat{G}^{R(A)}$ using the T-matrix approximation up to n_i-linear terms (in the Pauli basis) [10]:

$$\hat{\Sigma}^{R(A)} = n_i V_0^2 \hat{\gamma}^{(*)}(1 - V_0\hat{\gamma}^{(*)})^{-1} \equiv \mp i(\Gamma^{(*)}\hat{\sigma}_0 + \Gamma_z^{(*)}\hat{\sigma}_z) \tag{10}$$

$$\hat{\Sigma}^{<} = n_i V_0^2 \int \frac{d^2k}{(2\pi)^2}(1 - V_0\hat{\gamma})^{-1}\hat{G}^{<}(1 - V_0\hat{\gamma}^*)^{-1} \tag{11}$$

where $\hat{\gamma} = \int d^2k/(2\pi)^2 \hat{G}^R \equiv \gamma\hat{\sigma}_0 + \gamma_z\hat{\sigma}_z$, $\hat{G}^R = (\omega\hat{1} - \hat{H}_0 - \hat{\Sigma}^R)^{-1}$, with $\gamma = \gamma^r + i\gamma^i$, $\gamma_z = \gamma_z^r + i\gamma_z^i$, and calculated up to the lowest order:

$$\gamma^r = \frac{m}{4\pi}\ln\frac{\left|h^2 - \omega_F^2\right|}{k_0^4/4m^2} + \frac{\alpha^2\ln\left|k_+^2/k_-^2\right|}{\pi(k_+^2 - k_-^2)/m^3}; \quad \gamma^i = -\frac{\nu_- + \nu_+}{4}; \gamma_z^r = \frac{h\ln\left|k_+^2/k_-^2\right|}{\pi(k_+^2 - k_-^2)/m^2}; \gamma_z^i = \frac{h}{4}\left(\frac{\nu_+}{\lambda_+} - \frac{\nu_-}{\lambda_-}\right),$$

where $\nu_\pm = \frac{m\lambda_\pm}{\lambda_\pm \pm \alpha^2 m}$, $\nu_+ = 0$ when $\omega_F < h$, $\frac{\nu_+}{\lambda_+} = \frac{\nu_-}{\lambda_-}$ when $\omega_F > h$ and k_0 being the cutoff in the integration over the k vector. Note that we use the renormalizations $\omega_F \to \omega_F - \mathrm{Im}\Gamma$ and $h \to h - \mathrm{Im}\Gamma_z$ in Eqs. (9,11) which allows us to have purely imaginary self energies $\Sigma^{R(A)}$.

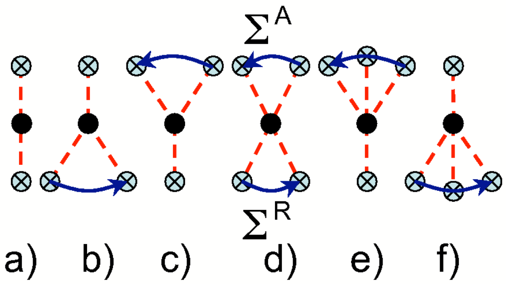

Figure 1: Diagrams representing the averaging procedure in calculating $\hat{\Sigma}^<$ in Eq. (11) where the upper part of the plot corresponds to $\hat{\Sigma}^A$ and the lower part corresponds to $\hat{\Sigma}^R$. The diagram a) leads to the side-jump contribution and the disorder-independent skew scattering [13], the diagrams b) and c) lead to conventional skew scattering, and the diagrams d), e) and f) lead to the higher order skew scattering.

However, real parts γ^r and γ_z^r still appear in $\hat{\Sigma}^<$ in Eq. (11).

In order to find the current in Eq. (7) up to zeroth order in V_0, we transform all elements of Eqs. (9,11) into the chiral basis and solve the kinetic equation up to zeroth order in V_0. That solution is used to solve the diagonal components of the kinetic equation up to the second order in V_0. Note that the expansion of $\hat{G}^{c<}_{+-/-+}$ starts from zero order terms in V_0 (see Eq. (6)) while the expansion of $\hat{G}^{c<}_{++/--}$ starts from terms proportional to V_0^{-2} which means that we only need to solve the non-diagonal components of the chiral kinetic equation up to zeroth order in V_0 while the diagonal components of the kinetic equation has to be solved up to the second order.

We find different components of $\hat{G}^{c<} = \widehat{G}^{c<}_{eq} + \widehat{G}^{c<}_{int} + \widehat{G}^{c<}_{(-2)} + \widehat{G}^{c<}_{sj} + \widehat{G}^{c<}_{sk}$ in the range (i) $-h < \omega_F < h$ when only " $-$ " chiral band is crossed by the Fermi level. The intrinsic contribution is already included in Eq. (8) and its Fermi level part is $\widehat{G}^{c<}_{int} = -iE\partial_\omega n_F A^-(\frac{\alpha\cos\varphi}{4\lambda_-}\hat{\sigma}_x - \frac{h\alpha\sin\varphi}{4\lambda_-^2}\hat{\sigma}_y)$. By solving the diagonal components of the Kinetic equation up to zeroth order in V_0, we obtain the part of $\hat{G}^{c<}$ proportional to V_0^{-2}:

$$\widehat{G}^{c<}_{(-2)} = -iE\partial_\omega n_F A^- \frac{4\lambda_-^2 k_- \sin\varphi}{n_i V_0^2 \kappa_-^2 \nu_-^2}\hat{\sigma}_{--},$$

where $\kappa_\pm = \sqrt{(\alpha k_\pm)^2 + 4h^2}$ and $\hat{\sigma}_{--/++} = (\hat{\sigma}_0 \pm \hat{\sigma}_z)/2$. By solving the non-diagonal components of the Kinetic equation up to zeroth order in V_0, we obtain the non-diagonal elements of the side-jump contribution:

$$\widehat{G}^{c<}_{sj} = \frac{E\partial_\omega n_F \alpha k_-^2}{\lambda_-}\left(\frac{(G_-^A + G_-^R)(\lambda_-\hat{\sigma}_y\cos\varphi - h\hat{\sigma}_x\sin\varphi)}{4\nu_-\kappa_-^2} + \frac{iA^-(\lambda_-\hat{\sigma}_x\cos\varphi + 3h\hat{\sigma}_y\sin\varphi)}{4\nu_-\kappa_-^2} + \frac{2iA^- h\cos\varphi\hat{\sigma}_{--}}{\kappa_-^2}\right) \quad (12)$$

while the diagonal contributions of side-jump are found by considering the diagram a) in Fig. 1 and by solving the diagonal components of the kinetic equation up to the second order in V_0. By considering the diagrams b)-f) in Fig. 1 and by solving the diagonal components of the kinetic equation up to the second order in V_0, we obtain the skew scattering contribution (the last term corresponds to the diagram a) and the disorder-independent skew scattering [13]):

$$\widehat{G}^{c<}_{sk} = iE\partial_\omega n_F A^-\left(\frac{8\Lambda}{n_i m V_0} - \frac{8\gamma^i}{n_i m V_0}\tan\varphi + \frac{32\alpha^2 k_-^2\gamma^r\gamma_z^i}{n_i\kappa_-^2} + \frac{3h\nu_-}{\lambda_-^2}\right)\frac{\alpha^2 k_-^3\lambda_-^2}{\kappa_-^4\nu_-^2}\hat{\sigma}_{--}\cos\varphi \quad (13)$$

where $\Lambda = (\frac{V_1^3}{V_0^3}\gamma_z^i + \frac{V_2^4}{V_0^3}(3\gamma^r\gamma_z^i + \gamma^i\gamma_z^r))$. Using Eq. (7), we arrive at the the Hall conductivity [14]:

$$\sigma_{xy} = \sigma_{xy}^{II} + \frac{e^2}{4\pi}\left(\frac{h\alpha^2\nu_-}{\lambda_-^2} - \frac{4hk_-^2\alpha^2}{\lambda_-\kappa_-^2} + \frac{3hk_-^4\alpha^2}{\kappa_-^4\nu_-} + \frac{8k_-^4\alpha^2\lambda_-^2}{n_i V_0\kappa_-^4\nu_-^2}(\Lambda + \frac{4k_-^2\alpha^2\gamma^r\gamma_z^i}{\kappa_-^2 m}\frac{V_1^6}{V_0^5})\right), \quad (14)$$

where $\sigma_{xy}^{II} = \frac{e^2}{4\pi}(1 - \frac{h}{\sqrt{\alpha^4+\lambda_F^2}})$ and $\lambda_F = \sqrt{h^2 + 2\alpha^2 m\omega_F}$. In Eqs. (13,14) we have made a straightforward generalization to a more general model of disorder: $V(\mathbf{r}) = \sum_i V_0^i\delta(\mathbf{r} - \mathbf{r}_i)$ with $\mathbf{r}_i$ random and strength distributions satisfying $\langle V_0^i\rangle_{dis} = 0$, $\langle (V_0^i)^2\rangle_{dis} = V_0^2$, $\langle (V_0^i)^3\rangle_{dis} = V_1^3$ and $\langle (V_0^i)^4\rangle_{dis} = V_2^4$. For the disorder described after Eq. (1), we have $V_0 = V_1 = V_2$ and for the white noise disorder we have $V_1 = 0$. Note that this result reduces to the Kubo formalism result of Ref. [10] when the last term bracket is calculated up to zeroth order in the strength of the disorder.

We repeat the same procedure in the range (ii) $h < \omega_F$ when both chiral bands are partially occupied. By using Eq. (7), within this limit we obtain that the intrinsic and side-jump contributions cancel each other, in agreement with Refs. 8, 9, 10, the Fermi sea contribution vanishes ($\sigma_{xy}^{II} = 0$) from Eq. (6), and the Hall conductivity is only non-zero for the higher order skew scattering arising from diagrams d)-f):

$$\sigma_{xy} = \frac{V_2^4}{n_i V_0^4}\frac{e^2 h\alpha^2\ln\left|\frac{k_-^2}{k_+^2}\right|}{\pi^2(k_-^2 - k_+^2)} \times \frac{\alpha^2 k_-^2 k_+^2(k_+^2 - k_-^2)/m + 2\sqrt{\alpha^4+\lambda_F^2}(k_+^4\lambda_- - k_-^4\lambda_+)}{16(\alpha^4+\lambda_F^2)^{3/2}(\lambda_F^2 - h^2)/\alpha^2} = -\frac{V_2^4}{n_i V_0^4}\frac{e^2 h\alpha^2\ln\left|k_-^2/k_+^2\right|}{\pi^2(k_-^2 - k_+^2)}. \quad (15)$$

This contribution from the higher order diagrams d)-f) was not considered in prior calculations within the Kubo formalism [8, 10] and only discussed without being calculated in Ref. 9. We have also used the numerical procedure of Onoda *et al.* [7] to verify this analytical result

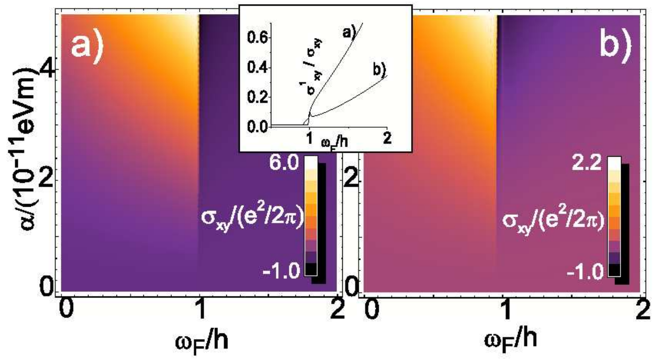

Figure 2: (Color online.) The anomalous Hall conductivity σ_{yx} vs. the Fermi energy ω_F and the spin-orbit coupling α (concentration of impurities $n_i = 10^{11}\mathrm{cm}^{-2}$, exchange splitting $h = 10$meV and $m^* = 0.05m$). The plot a) corresponds to the mobility $60\mathrm{m}^2/\mathrm{Vs}$ or weak impurity scattering strength while the plot b) corresponds to the mobility $12\,\mathrm{m}^2/\mathrm{Vs}$ and larger impurity strength. In the inset, σ^1_{xy} corresponds to the extrinsic skew scattering.

that identifies this new extrinsic regime in the 2DEG with Rashba. Although the contribution in this regime is proportional to $1/n_i$ it does not depend on V_0 as V_0^{-1}, as it is usual for the skew scattering. The result in Eq. (15) can be understood within the Boltzmann approach by writing the scattering rates in the chiral basis:

$$\omega_{\sigma\sigma'}(k,k') = \frac{2\pi}{\hbar} n_i T_c^R(k,k')_{\sigma\sigma'} T_c^A(k',k)_{\sigma'\sigma}\delta(\epsilon_k - \epsilon_{k'}), \tag{16}$$

where $\hat{T}_c^{R(A)}(k,k') = \hat{S}^\dagger(k)(1 - V_0\hat{\gamma}^{(*)})^{-1}\hat{S}(k')$. The asymmetric part with respect to k, k' in Eq. (16) is responsible for the skew scattering and is proportional to $n_i V_0^3 \Lambda$. Consequently, the Hall current should be proportional to

$$\sigma_{xy}^{\mathrm{skew}} \sim n_i V_0^3 \Lambda (\tau_\pm^{tr})^2 \sim \frac{n_i V_0^3(\gamma_z^i + V_0(3\gamma^r\gamma_z^i + \gamma^i\gamma_z^r))}{(n_i V_0^2)^2},$$

where $\tau_\pm^{tr}$ is the transport time for the $\pm$ chiral bands. The conventional skew scattering (V_0^{-1} order) appears due to the difference in the life-time for the $\pm$ chiral bands given by γ_z^i. However, for the Rashba model when both subbands are partially occupied we have $\gamma_z^i = 0$. In this limit, the asymmetry in the scattering still appears due to the difference in the Fermi energy renormalization for the $\pm$ chiral bands given by γ_z^r and leads to a V_0 independent contribution proportional to $1/n_i$.

In Fig. 2, we plot the anomalous Hall conductivity as a function of the Fermi energy ω_F and the spin-orbit coupling α for attractive impurities ($V_0 < 0$). We take typical parameters corresponding to a high quality 2DEG samples: the carrier concentration is in the range $10^{11}\mathrm{cm}^{-2}$, the maximum spin-orbit coupling is 5×10^{-11}eVm and the mobilities are 12 and $60\,\mathrm{m}^2/\mathrm{Vs}$. In the inset of Fig. 2 we analyze the importance of the extrinsic skew scattering caused by the impurity induced spin-orbit interaction $H_{SO} = \lambda(\boldsymbol{\sigma}\times\boldsymbol{\nabla}V)\cdot\mathbf{k}$ ($\lambda = 0.052\mathrm{nm}^2$ for GaAs [16]) that is always present in realistic systems. For the estimate we use the corresponding Hall conductivity [17], $\sigma^1_{xy} = \frac{e^2\lambda}{16 n_i V_0}\left(\nu_- k_-^4 - \nu_+ k_+^4\right)$. This conductivity becomes important for larger carrier concentrations and there should be a region of cross-over between the hybrid skew scattering and the extrinsic skew scattering (some interference between the two effects may take place). In the limit (i) ($\omega_F < h$), we observe skew scattering behavior ($\sigma_{xy} \sim 1/n_i V_0$) when the inverse Born scattering amplitude $\tau = 1/n_i V_0^2 m \gg 1/\epsilon_F$ (ϵ_F is the Fermi energy measured from the bottom of the band). For smaller τ, all curves have asymptotic behavior reaching a sum of side-jump and intrinsic contributions as it can be seen from Eq. (14) which represents the cross-over between the intrinsic-side-jump and extrinsic anomalous Hall effect. In the transition region to the limit (ii) ($\omega_F > h$), we observe a sudden drop of the Hall conductivity (see Fig. 2) with a sign change. The hybrid skew scattering should be observable in samples with dopants situated closer to the 2DEG to maximize the impurity strength as it can be seen from the inset of Fig. 2. Onoda *et al.* [7] analyze the region of $\tau\epsilon_F \sim 1$, finding $\sigma_{xy} \sim \sigma_{xx}^{1.6}$ scaling. This region is beyond applicability of our results which rely on the weak scattering limit $\tau\epsilon_F \gg 1$, since our approximations ignore the corrections to the conductivity $\sim 1/\tau\epsilon_F$ and the gradient expansion in this regime is not fully justified.

Summarizing, we analytically calculate the anomalous Hall current in a 2DEG ferromagnet with spin-orbit interaction using the Keldysh formalism. Complete agreement to the Kubo formula approach and to the Boltzmann equation approach is obtained. By considering the higher order skew-scattering diagrams, we are able to calculate a Hall current due to a hybrid skew scattering mechanism which is dominant when both subbands are partially occupied or when the system has white noise disorder. This particular Hall current does not depend on the impurity sign and strength.

We gratefully acknowledge discussions with V. Dugaev, J. Inoue, T. Jungwirth, A. H. MacDonald, Ar. Abanov, G.E.W. Bauer, N. Sinitsyn and S. Onoda. This work was supported by ONR under grant onr-n000140610122, by NSF under grant DMR-0547875 by SWAN-NRI and grants KJB100100802, LC510 and AV0Z10100521. J.S. is a Cottrell Scholar of the Research Foundation.

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