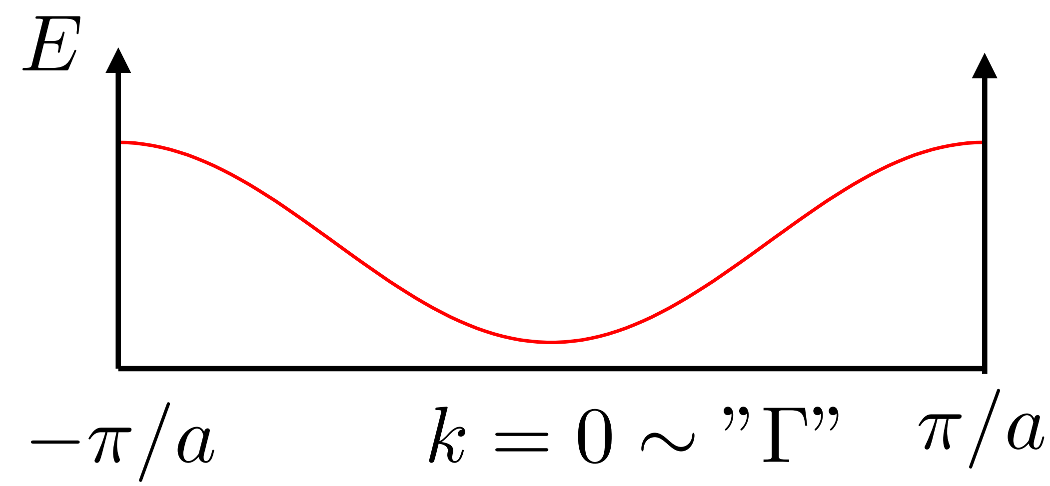
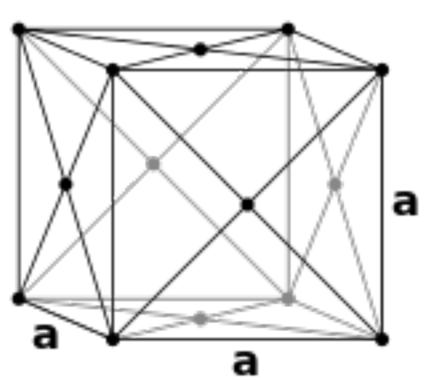
Band structure of a crystal

$\underbrace{E_0 \;—\; —}_{a} \;—\; —\; —$ $\longrightarrow$



$$E(k) = E_0 - 2|t| \cos ka$$



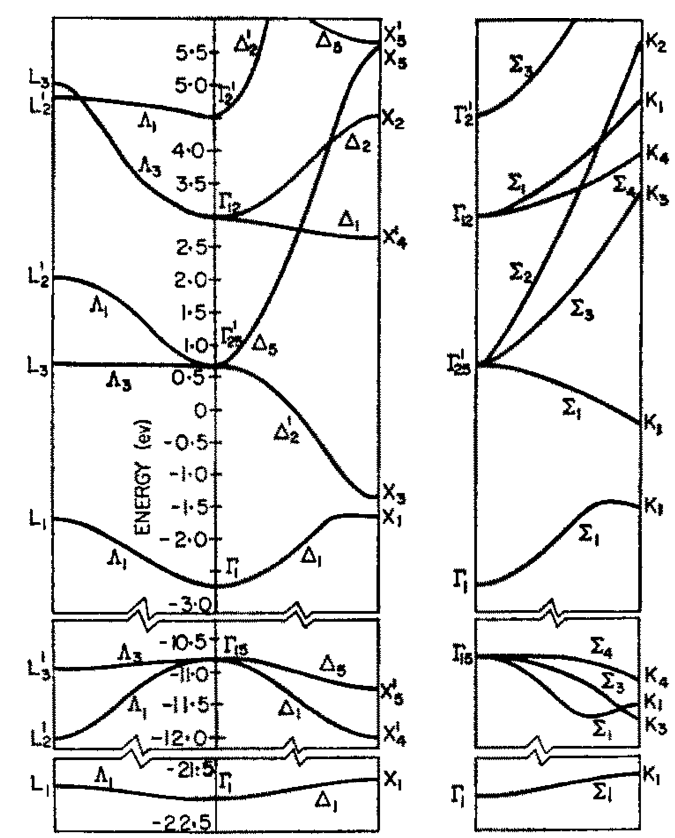
realistic example: Xe

- atoms = levels
- solid = bands

$6s$ — — — — — $\longrightarrow$

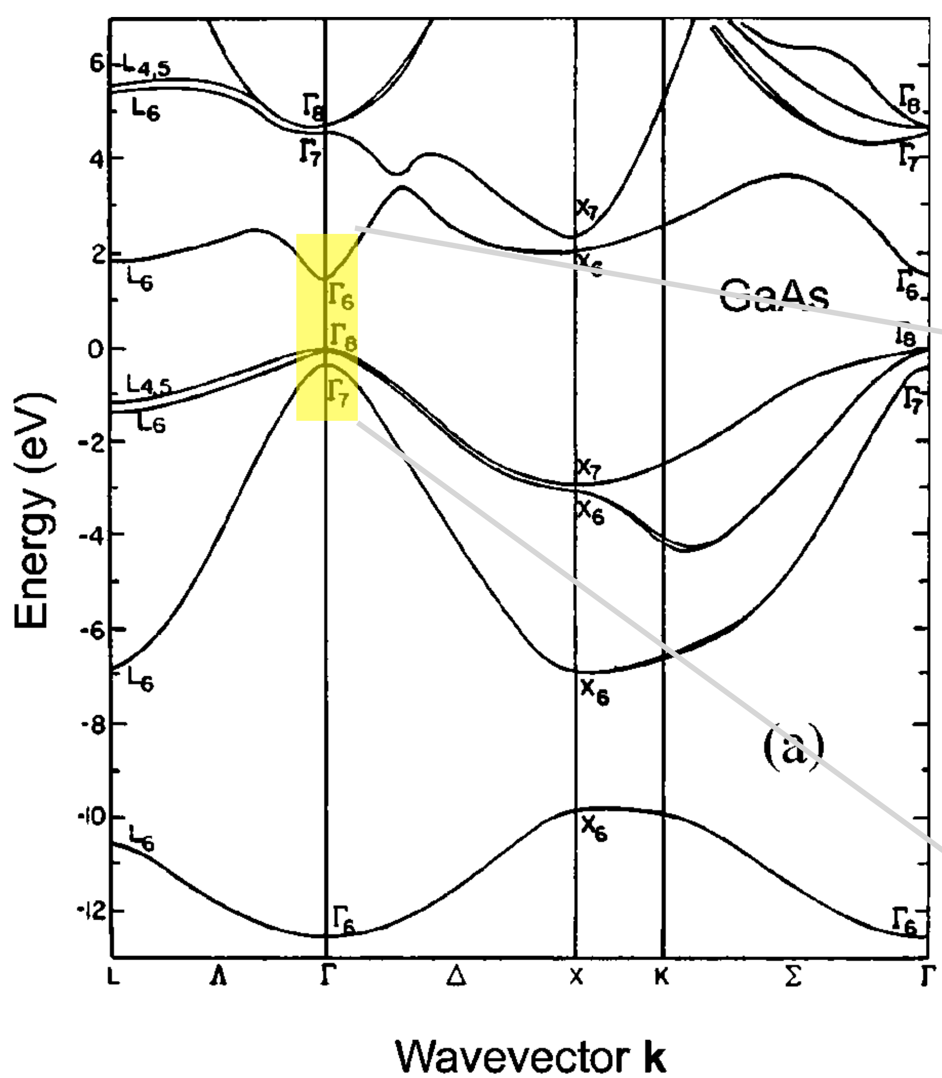
$5p$ — — — — — $\longrightarrow$

$5s$ — — — — — $\longrightarrow$

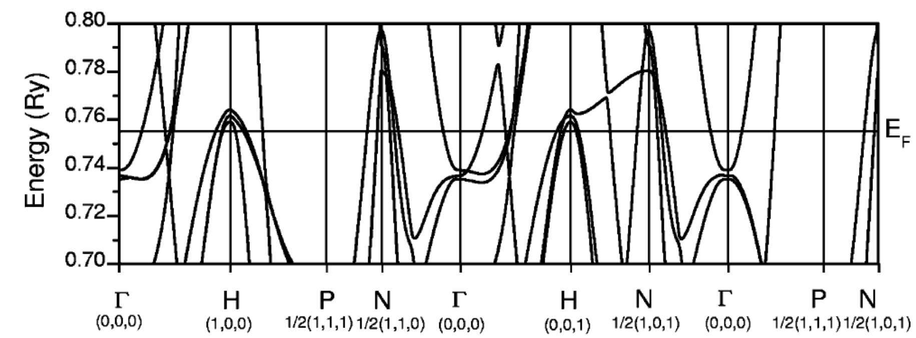


Typical band structures

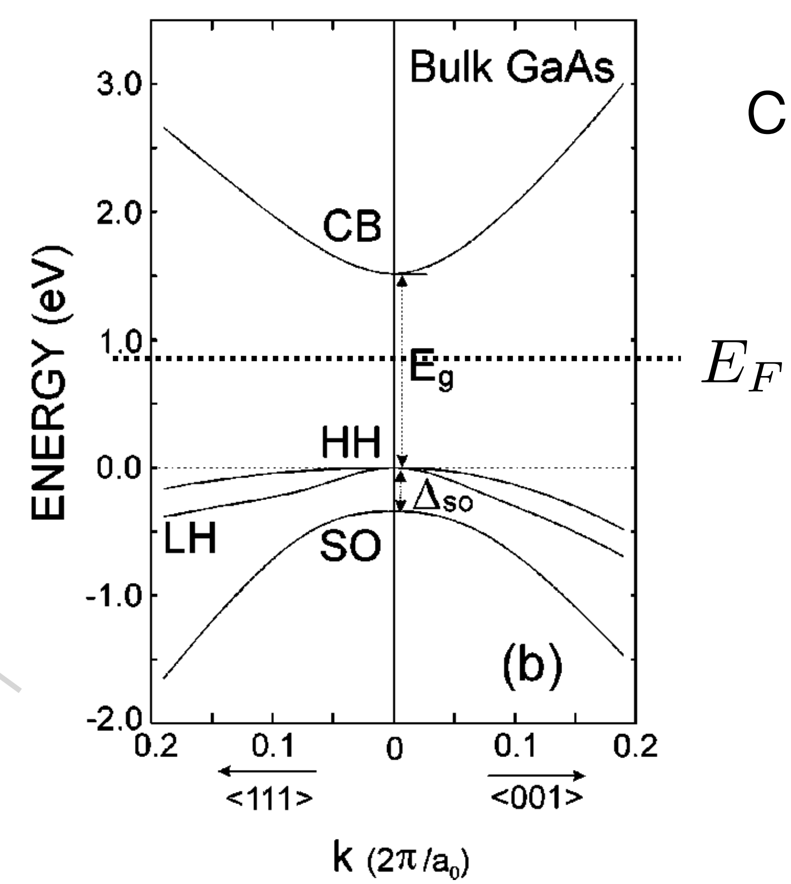
GaAs (semiconductor)



bcc iron (metal)



Fermi level

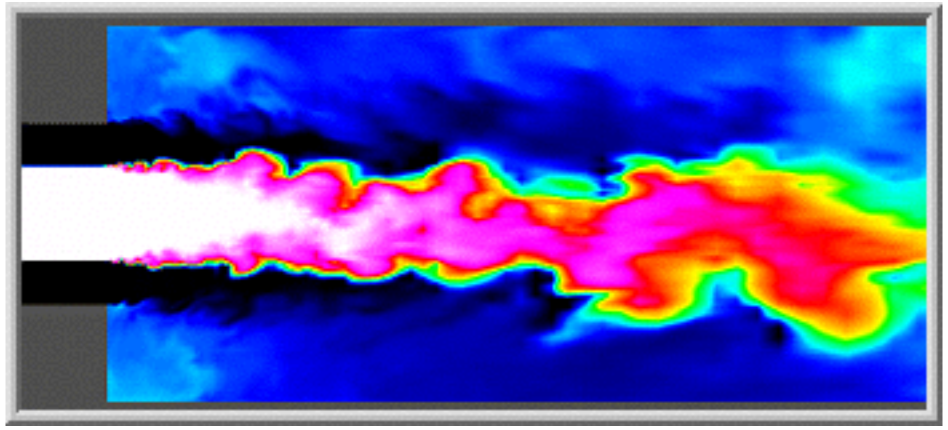


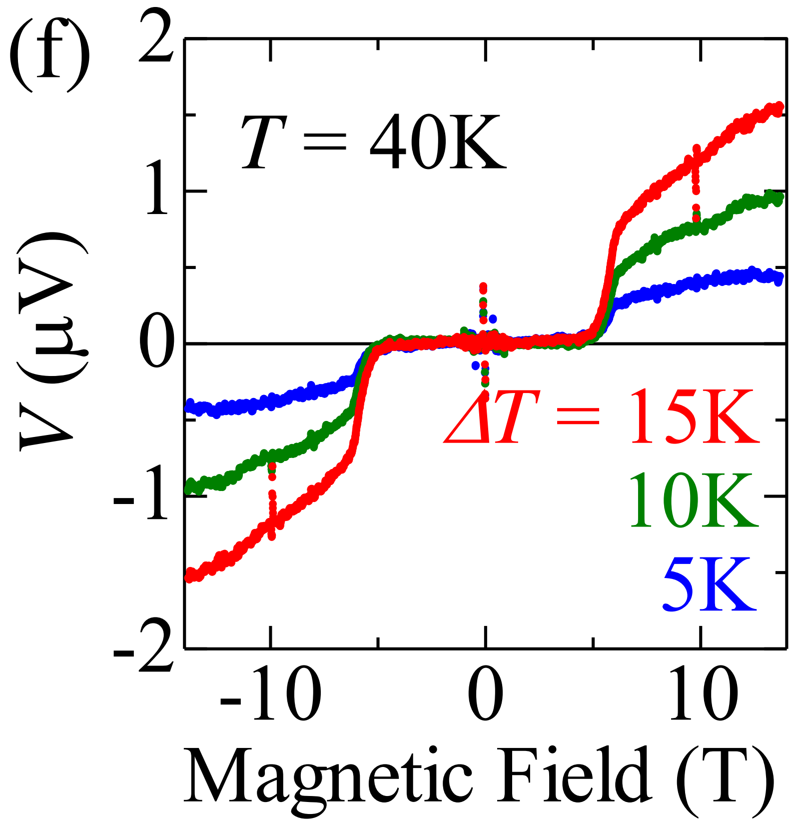
conduction band

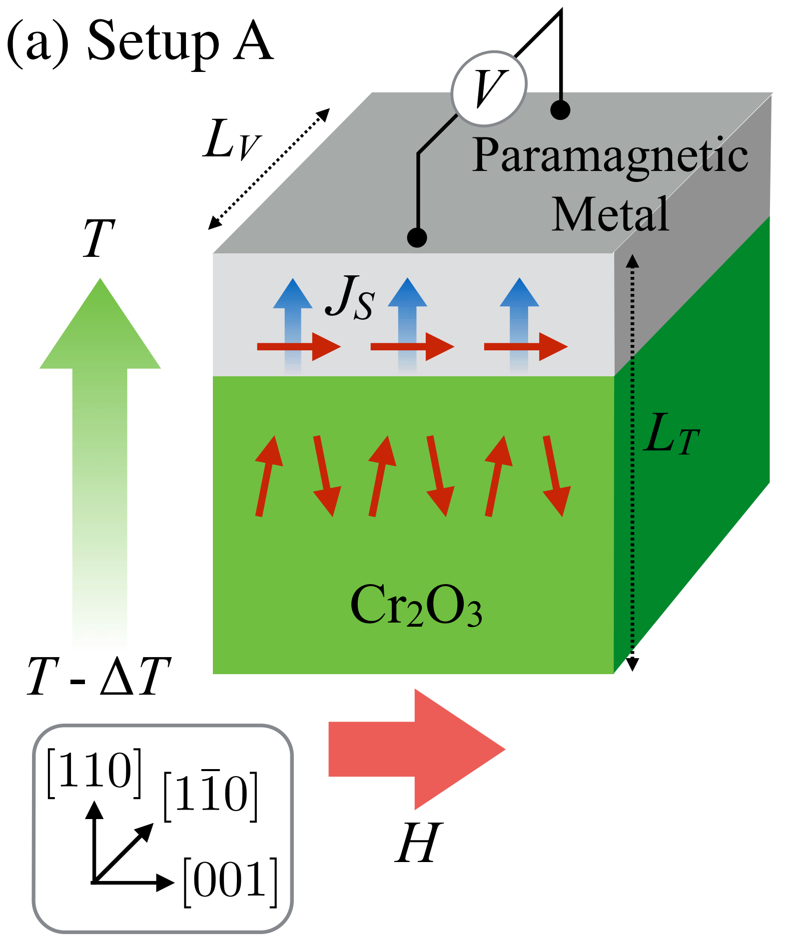
valence band

Transport

- of mass
- of charge
- of heat
- of spin
- …

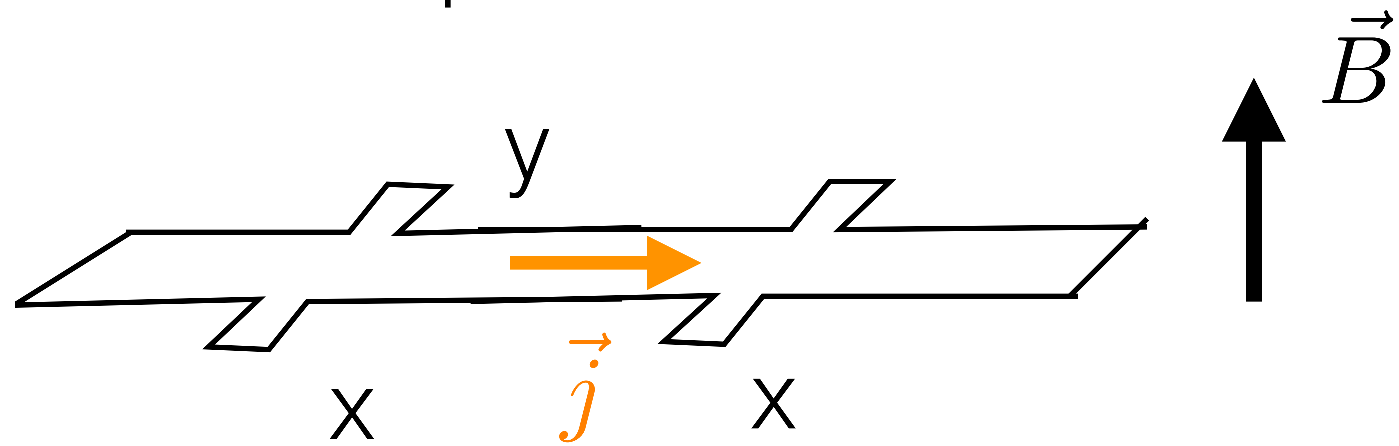




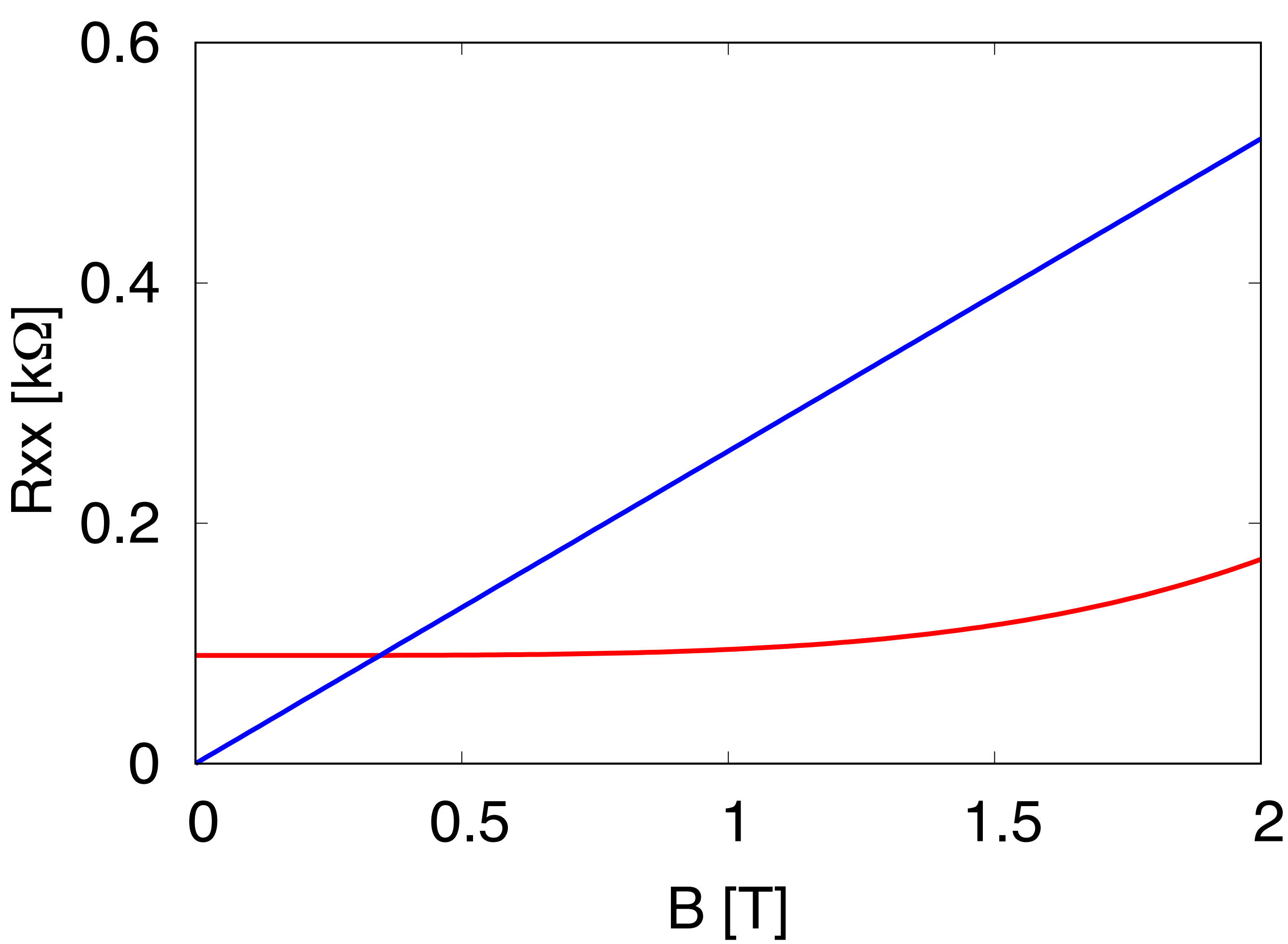


Transport: classical to quantum

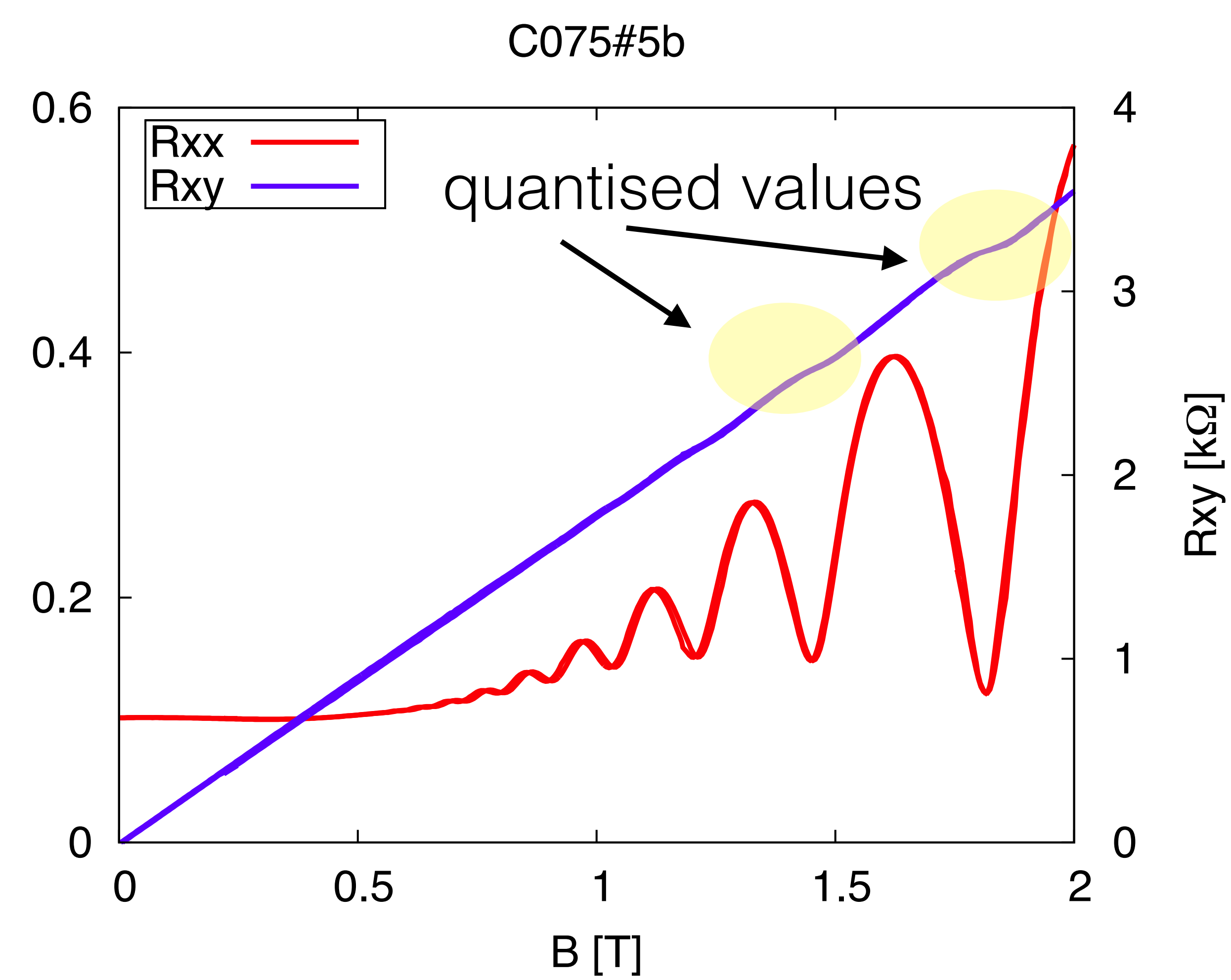
experimentalist's view



high temperature



low temperature

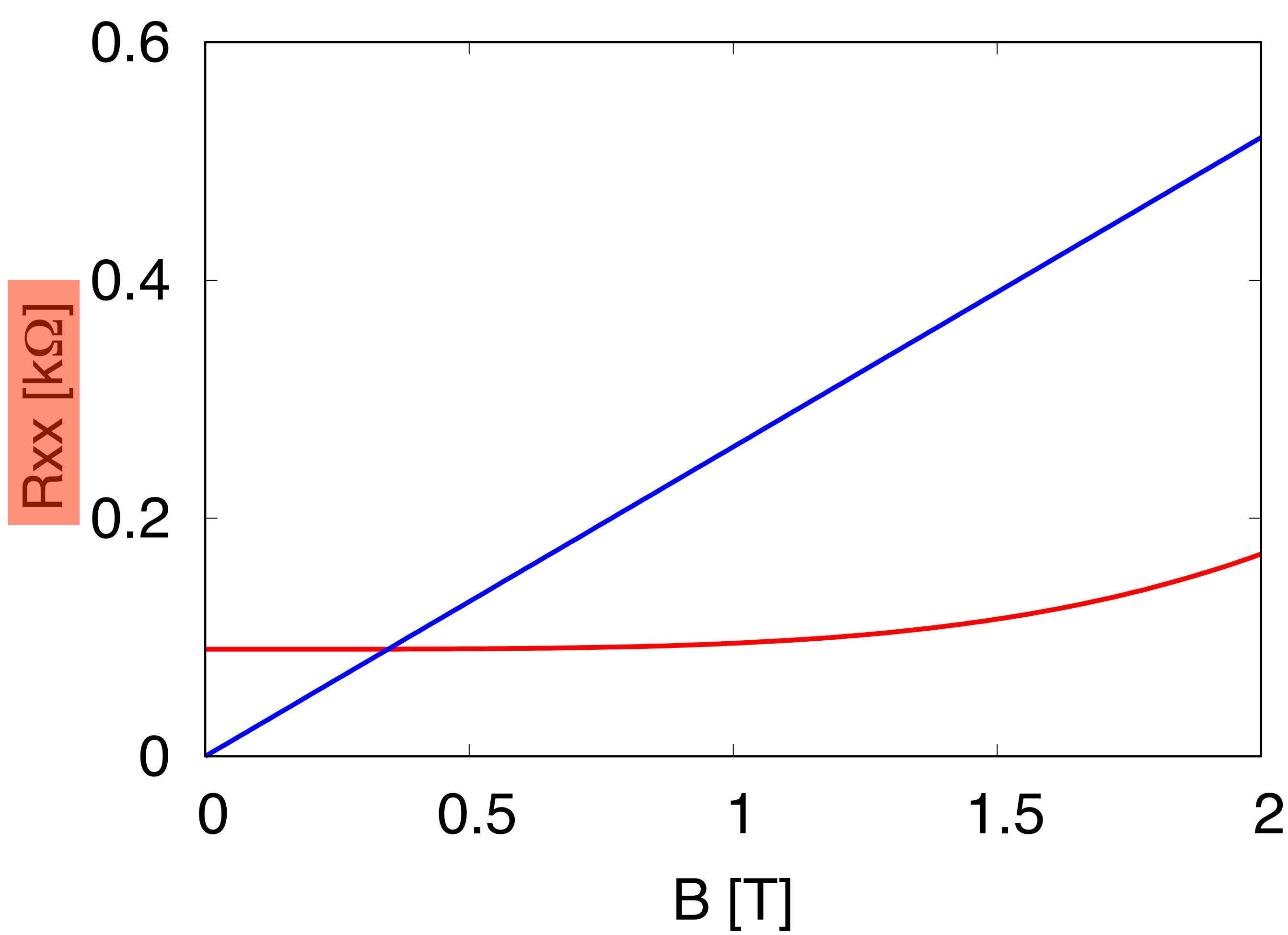


experimentalist's view

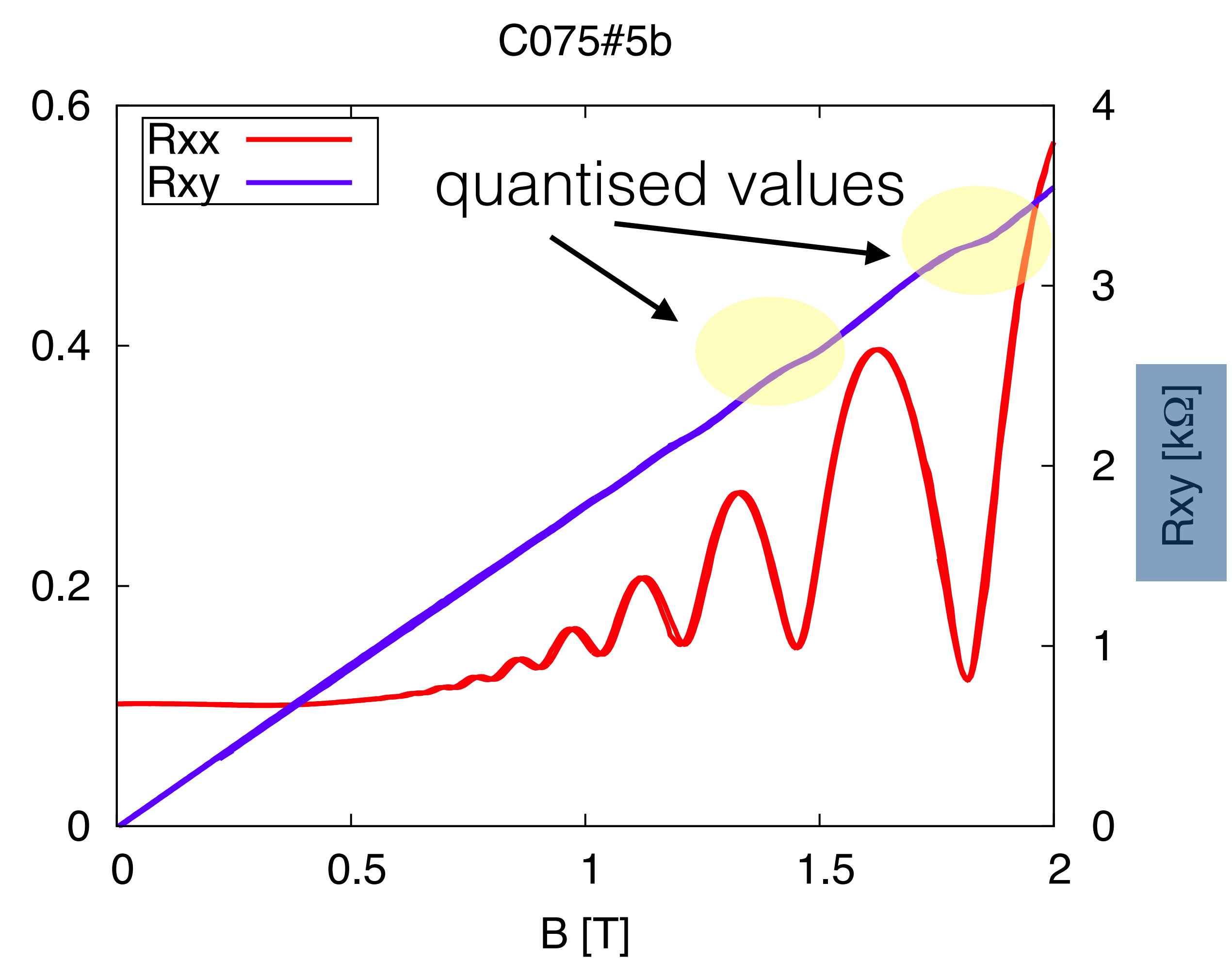
Transport: classical to quantum

$$\vec{E} = \overleftrightarrow{\rho}\,\vec{j} \qquad \overleftrightarrow{\rho} = \begin{pmatrix} \sigma_0^{-1} & 0 \\ 0 & \sigma_0^{-1} \end{pmatrix}$$

high temperature



low temperature



Approaches to electron transport

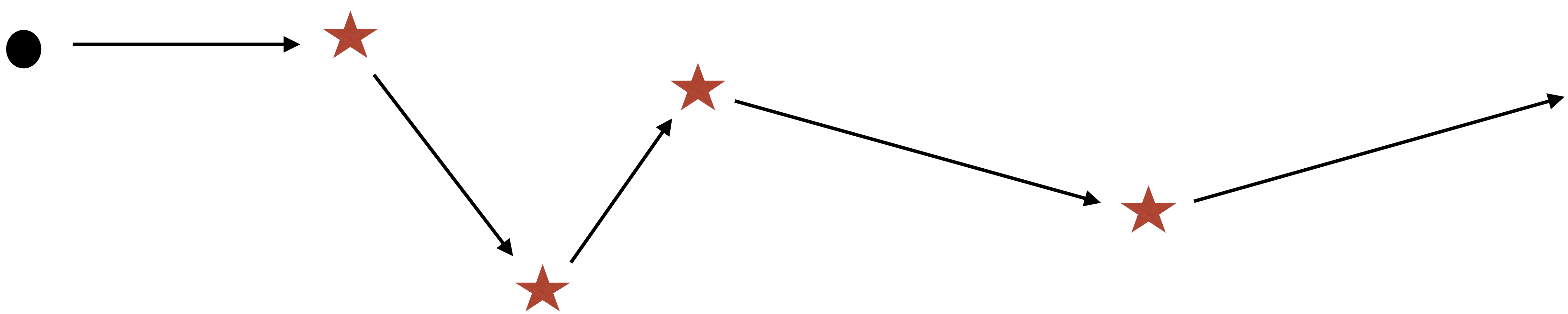
classical	semiclassical	quantum
$\vec{F} = m\vec{a}$	$\frac{\partial g}{\partial t} + \left(\frac{\partial g}{\partial t}\right)_{\mathrm{drift}} = \left(\frac{\partial g}{\partial t}\right)_{\mathrm{scatt}}$	$\frac{d\hat{f}(t)}{dt} + \frac{1}{i\hbar}\left[\hat{f}(t), \hat{H} + \hat{H}'(t)\right] + \hat{D}(\hat{f}, \hat{H}) = 0$
particle position and momentum	distribution function, Pauli exclusion principle	density matrix, operators

$$\sigma_0 = \frac{ne^2\tau}{m}$$

Drude formula for conductivity in an ideal electron gas where elastic collisions with impurities occur at rate $1/\tau$

Drude formula: classical derivation

$\vec{v}(t) = \vec{v}_0 + qt\vec{E}/m$ $\qquad t \mapsto \tau$ $\qquad \vec{j} = \frac{n}{m} e^2 \tau \vec{E}$

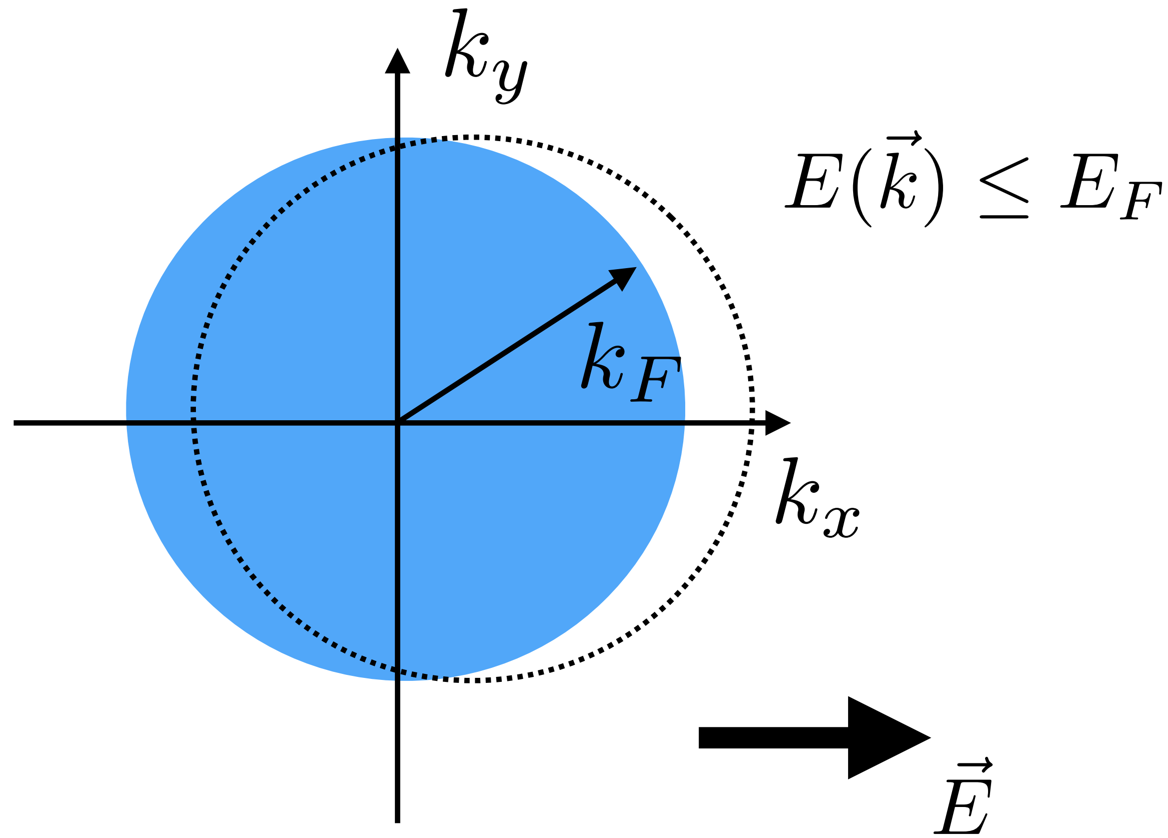


$\vec{j} = \sigma \vec{E}$, where $\sigma = \sigma_0$ $\qquad$ mobility: $\mu = e\tau/m$

Drude formula: semiclassical derivation

$$g = g(\vec{r}, \vec{k}, t)$$

$$E(\vec{k}) = \frac{\hbar^2}{2m}(k_x^2 + k_y^2)$$



determine what happens out of equilibrium

$$\vec{j} = \int \frac{d^2k}{(2\pi)^2} q\vec{v}(\vec{k}) g(\vec{k}) = q^2\tau \int \frac{d^2k}{(2\pi)^2} \left(-\frac{\partial f}{\partial \varepsilon}\right) \vec{v}(\vec{v}\cdot\vec{E})$$

using the solution of Boltzmann equation:

$$g(\vec{k}) = f(E(\vec{k})) + \Delta g(\vec{k})$$

$$\sigma_0 = \frac{ne^2\tau}{m}$$

Drude formula: fully quantum-mechanical derivation

$$\vec{j} \propto \mathrm{Tr}\ \hat{\vec{v}} \langle \hat{G} (\hat{\vec{v}} \cdot \vec{E}) \hat{G} \rangle$$

$$\sigma_{ij} \propto \mathrm{Tr}\ \hat{v}_i \langle \hat{G} \hat{v}_j \hat{G} \rangle$$

e.g. xx-component, finite frequencies:

$$\sigma(\omega) = \frac{\hbar e^2}{2\pi V} \int dE\ f(E) \Big[\mathrm{Tr}\ \big(G^+(E) - G^-(E) \big) v_x \frac{G^-(E+\hbar\omega) - G^-(E)}{\hbar\omega} v_x$$
$$- \mathrm{Tr}\ \big(G^+(E) - G^-(E) \big) v_x \frac{G^+(E) - G^+(E-\hbar\omega)}{\hbar\omega} v_x \Big]$$

what is the trace?

$$\mathrm{Tr}\ \hat{A} = \sum_i \langle \psi_i | \hat{A} | \psi_i \rangle$$

in general, need to know the wavefunctions but for free electrons:

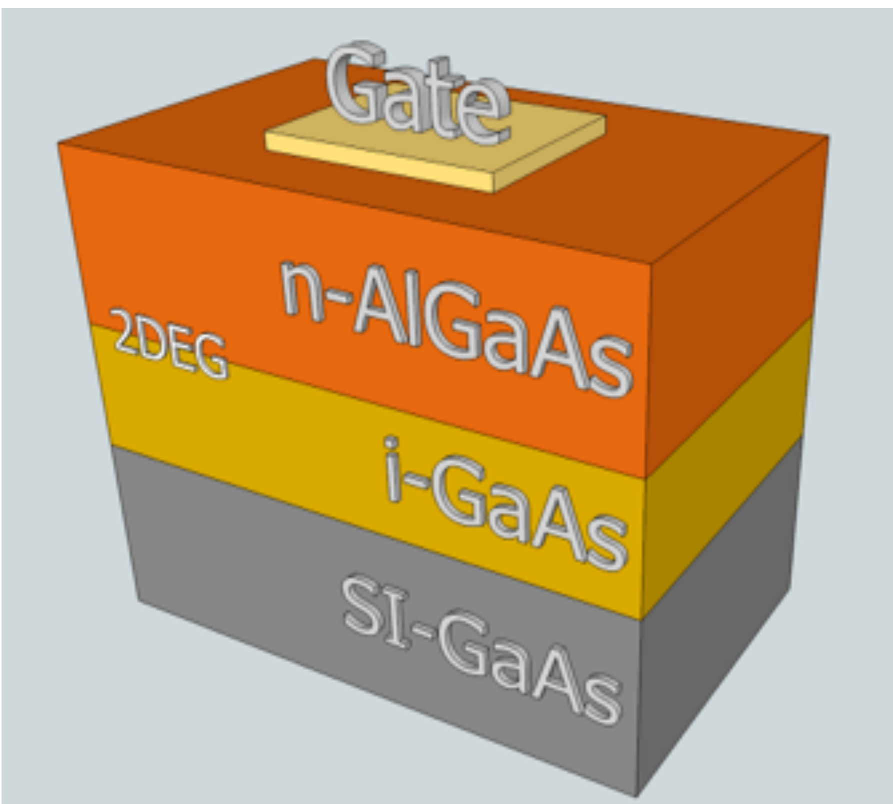
$$\langle x | \psi_k \rangle = e^{ikx} \ \Rightarrow \ \langle \psi_k | v_x | \psi_k \rangle = \hbar k / m$$

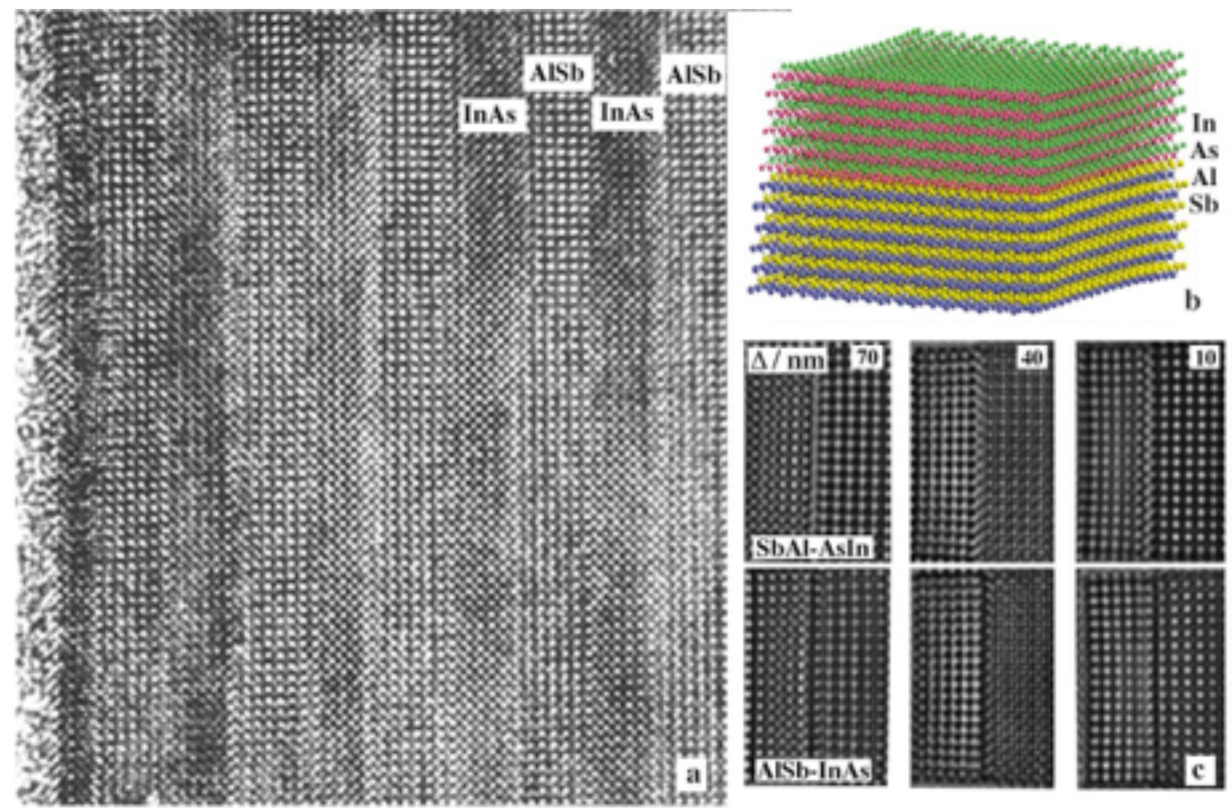
Examples of low-dimensional systems:

2D

semiconductor heterostructures

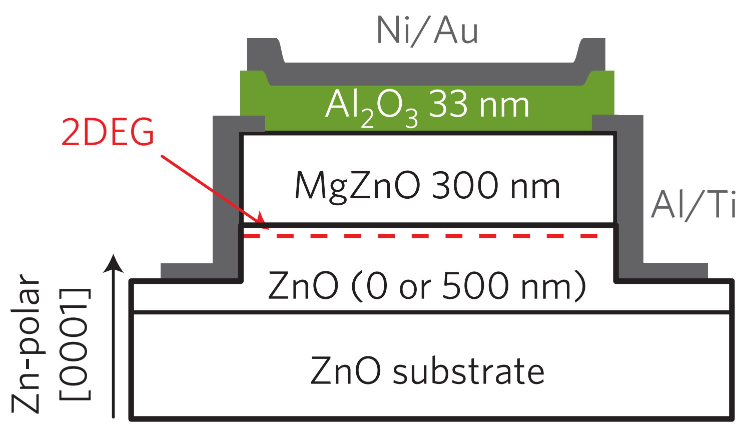
molecular beam epitaxy (MBE)

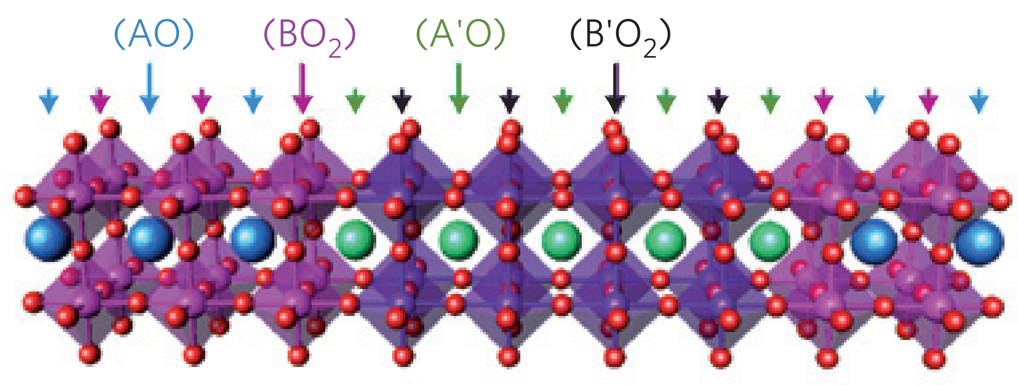




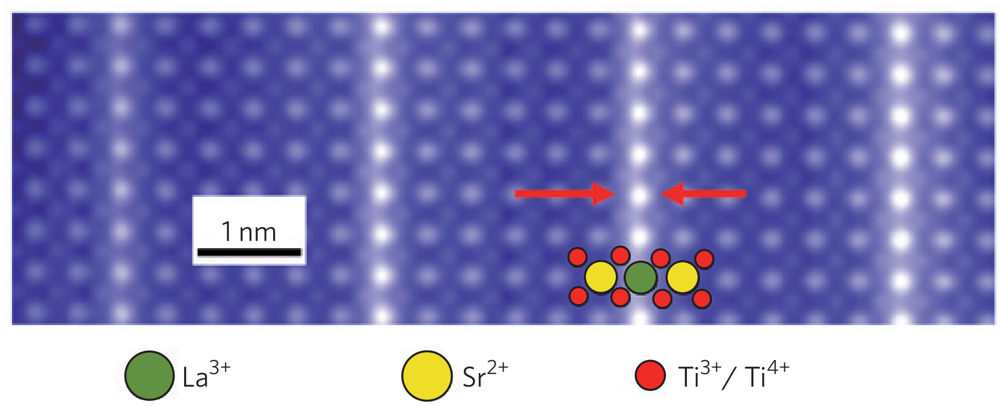
Examples of low-dimensional systems: 2D

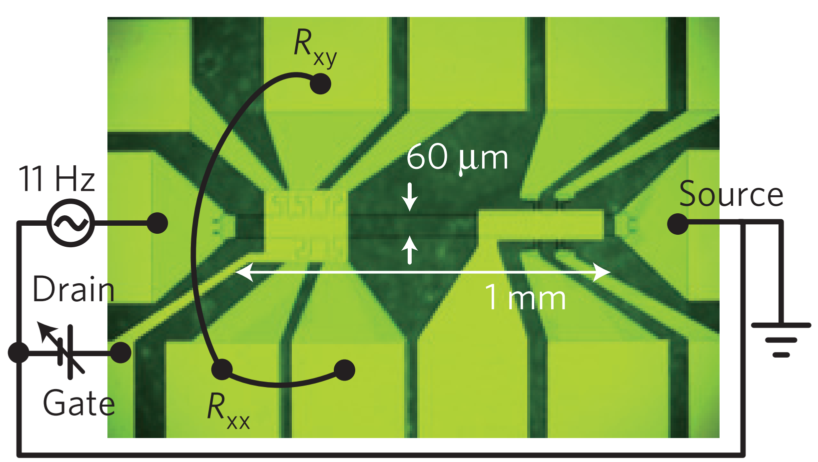
oxide heterostructures

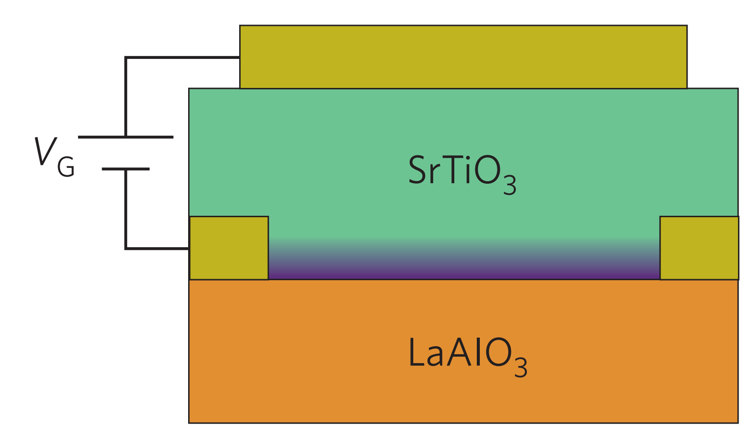




LTO in STO







Examples of low-dimensional systems: 2D

surface exposed to vacuum

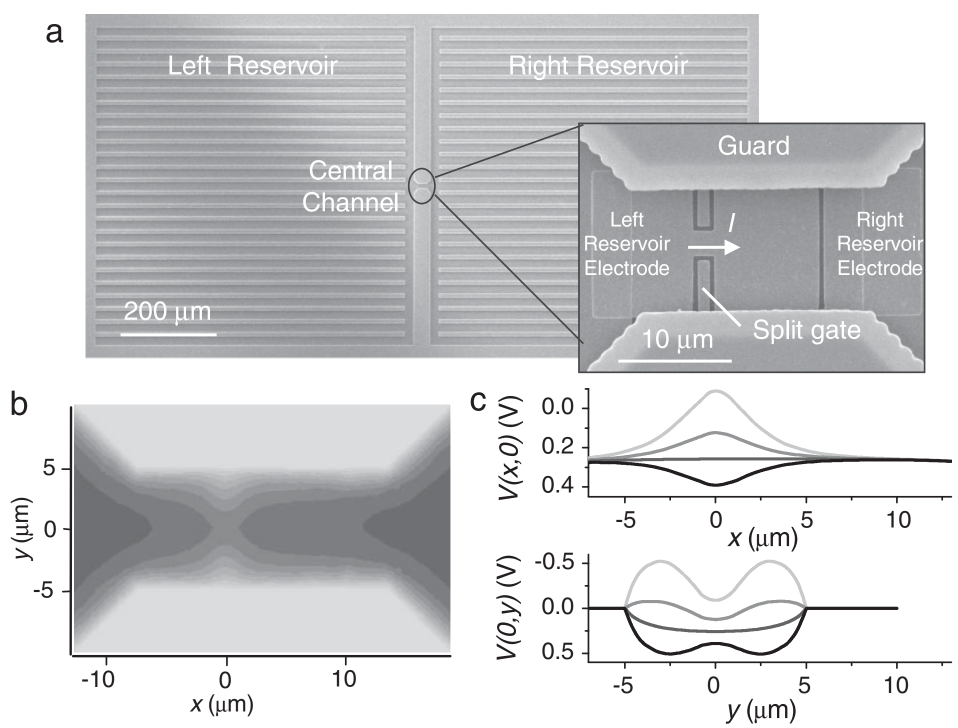


FIG. 1. (a) Scanning electron microscope image of sample 1. Inset: The split-gate electrode in the central channel. (b) Contour plot of the electrostatic potential at the helium surface in the central channel calculated by FEM software for $V_{gu} = V_{gt} = 0$ V, $V_r = +0.3$ V. The darker colors indicate regions of more positive potential (lower energy for electrons). (c) Calculated potential along the channel $V(x, 0)$ and across the constriction $V(0, y)$ for $V_{gu} = 0$ V, $V_r = +0.3$ V and $V_{gt} = +1.3$, $+0.3$, -0.7, -2.3 V (dark to light grey). Note that the vertical axes are inverted.

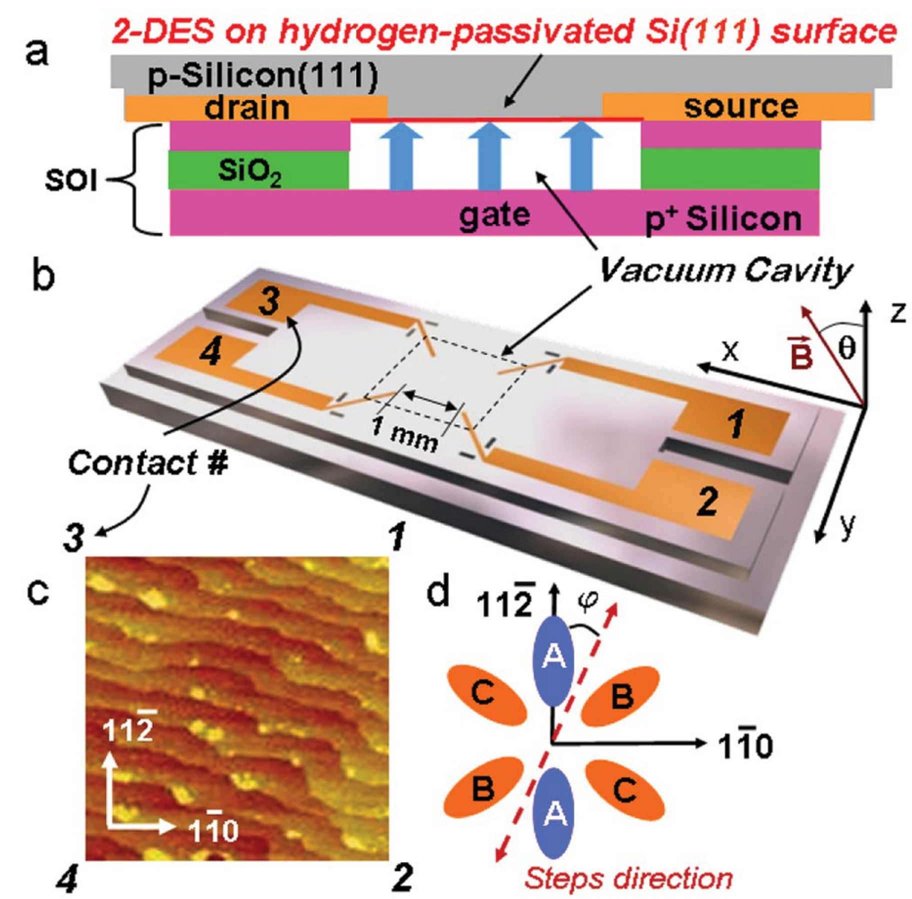


FIG. 1 (color). (a) Schematic cross-section of a H-Si(111) substrate contact bonded to a SOI substrate. A p^+ layer in the SOI defines the gate, where blue arrows depict the electric field. A 2DES is formed at the H-Si(111) surface within an encapsulated cavity. (b) The H-Si(111) substrate has four n^+ contacts numbered accordingly. Tilted magnetic fields are applied in the x-z plane. (c) A 1 μm × 1 μm AFM image of atomic steps on a H-Si(111) surface in relation to the crystal directions and the contacts of the device. (d) The projection of the six valleys for the Si(111) surface with pairs of valleys labeled A, B, and C.

Examples of low-dimensional systems: 2D

graphene

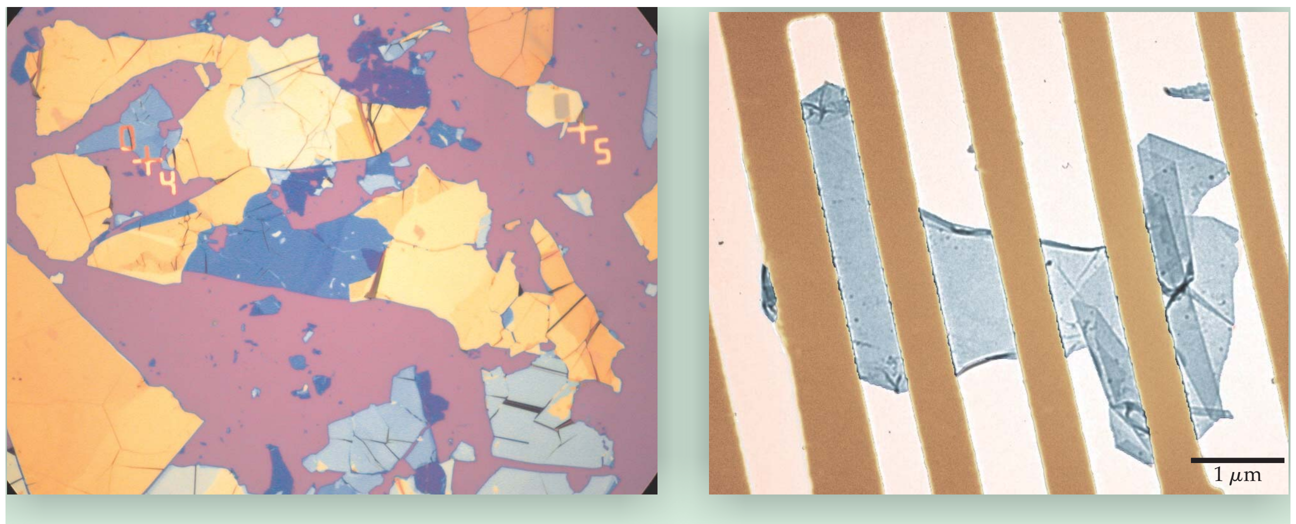
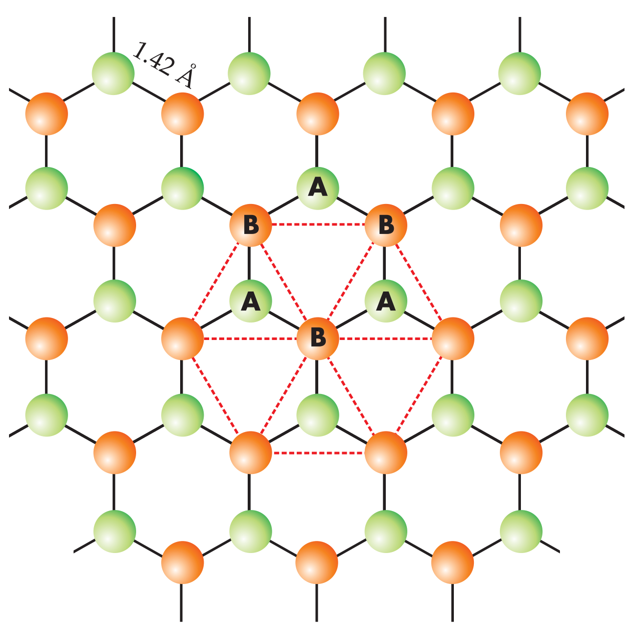


Figure 1. Spotting graphene. **(a)** Different colors in this 300-micron-wide optical micrograph reveal the presence of graphite flakes with differing thicknesses rubbed from bulk graphite onto the surface of an oxidized silicon wafer. Individual atomic planes are hidden in the debris but still can be found by zooming in and searching for flakes that show the weakest contrast. Force microscopy is used later to measure the thickness of identified crystallites. **(b)** A one-atom-thick single crystal of graphene hangs freely on a scaffold of gold wires, as seen with a transmission electron microscope. (Adapted from ref. 12.)



Wafer-Scale Graphene Integrated Circuit

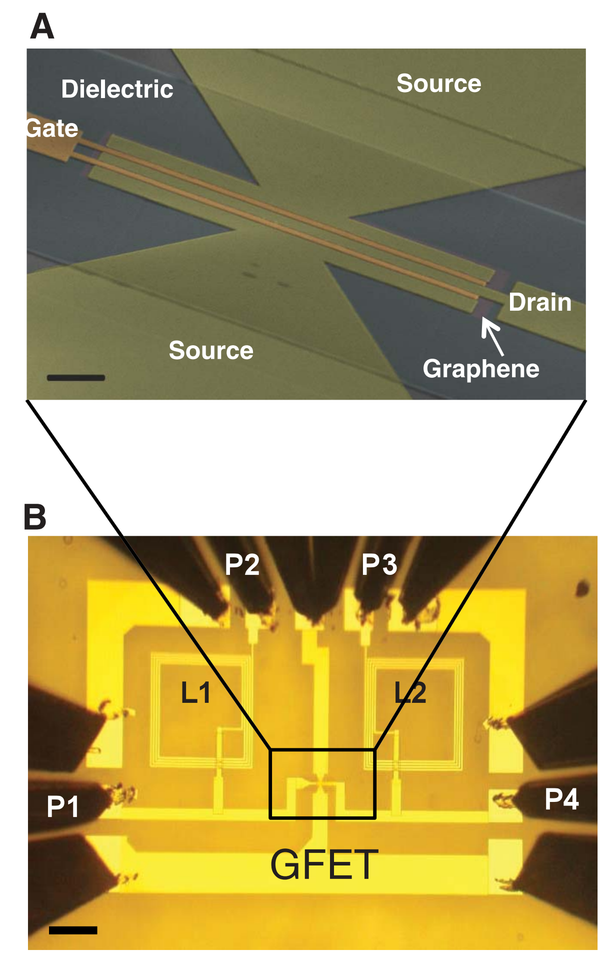
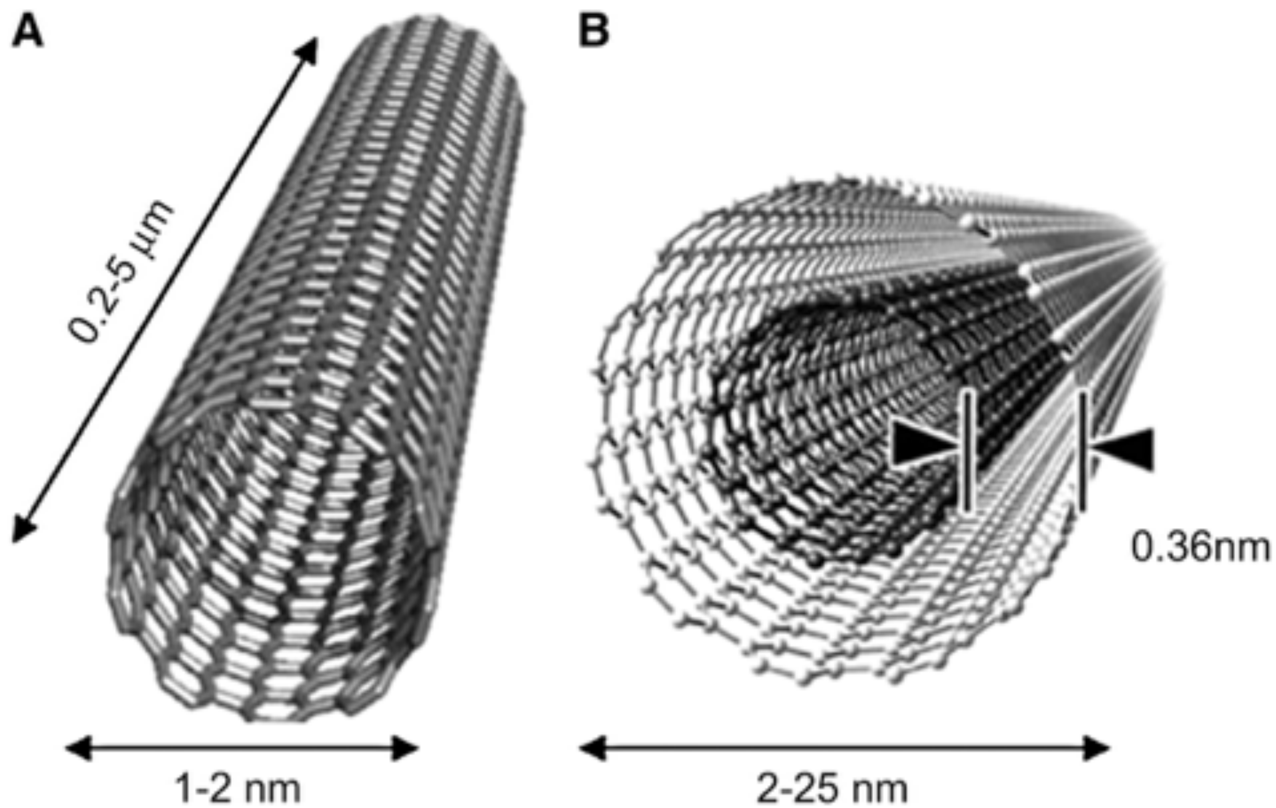
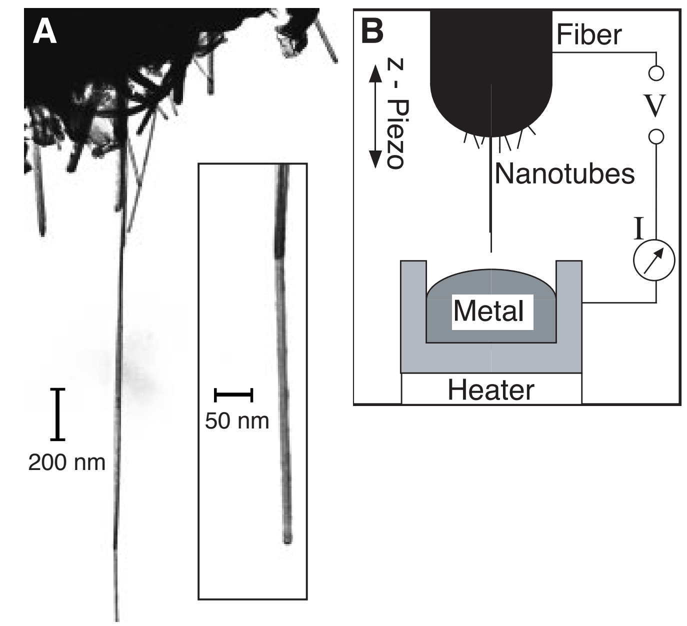


Fig. 2. Images of graphene ICs. (**A**) Scanning electron image of a top-gated, dual-channel graphene transistor used in the mixer IC. The gate length is 550 nm and the total channel width, including both channels, is 30 μm. Scale bar, 2 μm. (**B**) Optical image of a completed graphene mixer including contact pads. The gound-signal-ground configuration is implemented for the probe pads suitable for direct RF testing. Scale bar, 100 μm.

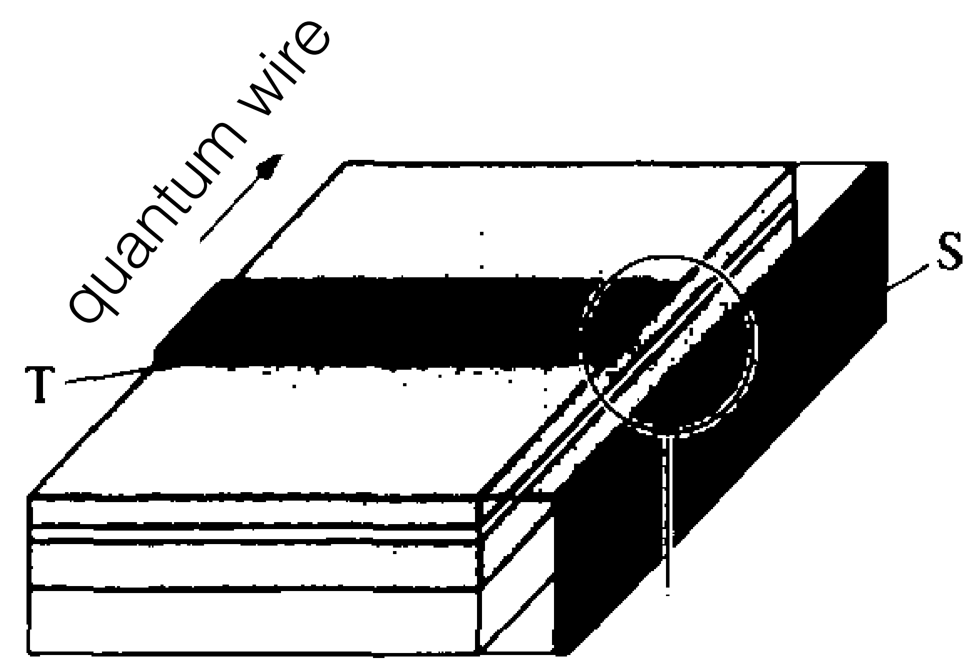
Examples of low-dimensional systems: 1D

carbon nanotubes, narrow channels





cleaved edge overgrowth



Examples of low-dimensional systems: 0D

semiconductor quantum dots

Colloidal CdSe nanocrystals (NC), diameter 1.7-4.5 nm (left to right) under UV illumination.

Magnetically ordered nanocrystals (Mn-doped CdSe), optical control of magnetism